

String predictions for near future colliders from one-loop scattering amplitudes around D-brane worlds

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ABSTRACT: We derive compact expressions for one-loop scattering amplitudes of four open-string vector bosons around supersymmetric configurations with intersecting or magnetized D-branes on toroidal orbifolds. We check the validity of our formulae against the structure of their singularities and their behaviour under modular transformations to the transverse channel, exposing closed string exchange. We then specialize to the case of forward scattering and compute the total cross section for two massless open string vector bosons on the brane to decay into closed strings in the bulk, relying on the optical theorem. Although not directly related to collider signatures our predictions represent a step forward towards unveiling phenomenological implications of open and unoriented superstrings.

KEYWORDS: Conformal Field Models in String Theory, Intersecting branes models, Superstring Vacua, D-branes.

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1. Introduction and conclusions

Vacuum configurations with open and unoriented strings have proven to be a particularly rich arena where to address compelling phenomenological issues in a string context amenable to explicit computations (see e.g. [1–6] for comprehensive reviews).

After their systematization [7–15], including subtle effects such as rank reduction induced by a quantized NS-NS antisymmetric tensor [14–17] and the minimal coupling of RR p-form potentials in an asymmetric superghost picture [14, 15], these theories have received an enormous boost when their geometric description in terms of D-branes and Ω -planes [18, 19], pioneered in [20, 21], has catalyzed the attention of the community.

Compactifications on toroidal orbifolds [22–27] with intersecting [28–32] and/or magnetized [33–37] branes represent a simple yet interesting class of models where problems connected with the presence of (large) extra dimensions [38–40], supersymmetry breaking [41–45] and moduli stabilization [46–55] can be tackled in a controllable way. The study of interactions that determine the structure of the low energy effective action and higher derivative corrections thereof has lead to enormous effort at tree (disk and sphere) level [56–61] but relatively little is known at one-loop [62–66] and beyond [67]. Aim of the present investigation is to improve the situation in view of potential phenomenological application of this kind of analyses in formulating predictions for near future colliders based on models with relatively small string tension and /or large extra dimensions [68–71].

With this goal in mind, we use the standard NSR formalism [72] to derive compact expressions for one-loop scattering amplitudes of four open-string vector bosons around vacuum configurations with open and unoriented strings preserving some supersymmetry. Remarkably, in addition to the standard elliptic functions, our final formulae only involve two more modular forms denoted by $\mathcal{E}_{\mathcal{N}}$ and $\mathcal{F}_{\mathcal{N}}$ in the following. We check the validity of our results, that generalize and extend the classic results of Green and Schwarz in $D = 10$ [73, 74], against the structure of their singularities and the properties under modular transformations to the transverse channel that exposes closed string tree-level exchange.

We then compute the total cross section $\sigma_{tot}(s)$ for two (massless) open string vector bosons on the brane to decay into closed strings in the bulk around general unoriented vacuum configurations preserving at least $\mathcal{N} = 1$ supersymmetry. The optical theorem relates $\sigma_{tot}(s)$ to the imaginary part of the forward scattering amplitude $\mathcal{A}(s)$. We work to lowest order in g_s , i.e. $|\text{disk}|^2 \approx \text{Im}(\text{annulus})$. In principle, one could directly compute the amplitudes for the decay of two massless open string states into (massive) closed string states on the disk. These amplitudes are however plagued with subtle normalization problems and we find it more convenient to extract them from the non-planar one-loop forward scattering amplitudes [68–71].

The relevant contribution to the process corresponds to an amplitude with vector bosons with the same Chan-Paton factors that can annihilate into gauge-singlet closed string states. More general initial states, involving massless open string scalars or fermions can be computed similarly. Processes initiated by gauginos, in the Adjoint of the Chan-Paton group, or matter scalars, in chiral multiplets, are not relevant for collider physics. In

addition to the presently studied processes initiated by vector bosons, transforming in the Adjoint of the Chan-Paton group and belonging to the untwisted sector of the orbifold with integer modes connecting parallel or equally magnetized branes, one should also consider processes initiated by matter fermions, that may either belong to the untwisted sector or to twisted sectors connecting branes intersecting at angles or with different magnetic fluxes. The latter can be studied in parallel and will be the focus of [75].

Our present analysis exploits remarkable properties of elliptic functions [76] and free field propagators on genus one surfaces with or without boundaries and crosscaps to compute and simplify the worldsheet correlators. After deriving compact expressions for the one-loop non-planar amplitudes under consideration, we specialize them to the case of forward scattering. Extracting their imaginary part, we determine the total cross section for the decay into closed strings as a function of the remaining Mandelstam variable $s = -(p_1 + p_2)^2 = -2p_1 \cdot p_2$. It shows the expected peak and threshold structure that encodes the properties of the brane configuration in the internal space.

The plan of the paper is as follows. In section 2 we recall basic formulae for vertex operators and tree level scattering amplitudes. In section 3 we compute the one-loop contractions relevant for CP even processes. Section 4 contains a summary of the results for the reader who is not interested in the details of the derivations. CP odd amplitudes receiving contribution only from $\mathcal{N} = 1$ sectors are discussed in section 5. The case of forward scattering both for CP even and CP odd amplitudes is studied in section 6, while the total cross sections for the decay into closed strings in the bulk are discussed in section 7. We present our final comments and draw perspectives for our future investigation in section 8.

Some properties of elliptic functions are collected in the appendices that should be consulted for notation and conventions.

2. Tree level amplitude

For completeness, comparison and later purposes we report here the results for the tree level (disk) scattering amplitude of four open string gauge bosons. The result is independent of the amount of supersymmetry enjoyed by the vacuum configuration, since the relevant vertex operators in the NSR formalism¹

$$\begin{aligned} V_0^{a\tilde{a}}(z) &= a^\mu (\partial X_\mu + ip \cdot \psi \psi_\mu) e^{ip \cdot X}(z) T_{CP}^{a\tilde{a}}, \\ V_{-1}^{a\tilde{a}}(z) &= a^\mu \psi_\mu e^{-\varphi} e^{ip \cdot X}(z) T_{CP}^{a\tilde{a}}, \end{aligned} \tag{2.1}$$

with $p^2 = 0$, $a \cdot p = 0$ for BRS invariance and $\tilde{a} = \Omega a$, do not depend on the details of the compactification encoded in the internal CFT, to be specified later on. In other words they only involve the identity operator that has trivial correlators. The matrices $T_{CP}^{a\tilde{a}}$ belong to the adjoint of the Chan-Paton group, i.e. to the $\mathbf{N}\bar{\mathbf{N}}$ for $U(N)$ or to the $\mathbf{N}(\mathbf{N} \pm \mathbf{1})/2$ for $Sp(N)$ or $SO(N)$.

¹Unless otherwise stated, we set $\alpha' = 1/2$ henceforth.

Vertex operators are inserted on the boundary of the disk that is conformally equivalent to the upper half plane whereby $z_i = x_i \in \mathbb{R}$. Three c ghost insertions are needed to fix $SL(2, \mathbb{R})$ invariance. The tree level four bosons scattering amplitude is then given by

$$\mathcal{A}_{vv \rightarrow vv}^{tree}(p_i, a_i) = g_s \text{tr}(T_1 T_2 T_3 T_4) \int dz_3 \langle cV_0(z_1; p_1, a_1) cV_{-1}(z_2; p_2, a_2) V_0(z_3; p_3, a_3) cV_{-1}(z_4; p_4, a_4) \rangle, \quad (2.2)$$

up to permutations of the external legs.

After performing the free field contractions and including the relevant non-cyclic permutations, one finds²

$$\begin{aligned} \mathcal{A}_{vv \rightarrow vv}^{tree}(p_i, a_i) &= g_s (2\pi)^4 \delta\left(\sum_i p_i\right) K_{vv \rightarrow vv}^{tree}(p_i, a_i) \times \\ &\{[\text{tr}(T_1 T_2 T_3 T_4) + \text{tr}(T_1 T_4 T_3 T_2)]B(s, t) + \\ &[\text{tr}(T_1 T_3 T_4 T_2) + \text{tr}(T_1 T_2 T_4 T_3)]B(t, u) + \\ &[\text{tr}(T_1 T_4 T_2 T_3) + \text{tr}(T_1 T_3 T_2 T_4)]B(u, s)\}, \end{aligned} \quad (2.3)$$

where

$$B(s, t) = \int_0^1 dx x^{2\alpha' p_1 \cdot p_2 - 1} (1-x)^{2\alpha' p_1 \cdot p_3 - 1} = \frac{\Gamma(-\alpha' s) \Gamma(-\alpha' t)}{\Gamma(\alpha' u)} \quad (2.4)$$

is Euler Beta function that appears in the celebrated Veneziano amplitude, $s = -(p_1 + p_2)^2 = -2p_1 \cdot p_2$, $t = -(p_1 + p_4)^2 = -2p_1 \cdot p_4$, $u = -(p_1 + p_3)^2 = -2p_1 \cdot p_3$ and thus $s + t + u = 0$.

The bosonic kinematic factor $K_{vv \rightarrow vv}^{tree}(p_i, a_i)$ is totally symmetric (not simply cyclically symmetric!) and reads [73, 74]

$$\begin{aligned} K_{vv \rightarrow vv}^{tree}(p_i, a_i) &= -\frac{1}{4} (s t a_1 \cdot a_3 a_2 \cdot a_4 + u s a_1 \cdot a_4 a_2 \cdot a_3 + t u a_1 \cdot a_2 a_3 \cdot a_4) \\ &+ \frac{s}{2} (a_1 \cdot p_4 a_3 \cdot p_2 a_2 \cdot a_4 + a_2 \cdot p_3 a_4 \cdot p_1 a_1 \cdot a_3 + a_1 \cdot p_3 a_4 \cdot p_2 a_2 \cdot a_3 + a_2 \cdot p_4 a_3 \cdot p_1 a_1 \cdot a_4) \\ &+ \frac{t}{2} (a_2 \cdot p_1 a_4 \cdot p_3 a_3 \cdot a_1 + a_3 \cdot p_4 a_1 \cdot p_2 a_2 \cdot a_4 + a_2 \cdot p_4 a_1 \cdot p_3 a_3 \cdot a_4 + a_3 \cdot p_1 a_4 \cdot p_2 a_2 \cdot a_1) \\ &+ \frac{u}{2} (a_1 \cdot p_2 a_4 \cdot p_3 a_3 \cdot a_2 + a_3 \cdot p_4 a_2 \cdot p_1 a_1 \cdot a_4 + a_1 \cdot p_4 a_2 \cdot p_3 a_3 \cdot a_4 + a_3 \cdot p_2 a_4 \cdot p_1 a_2 \cdot a_1) \end{aligned} \quad (2.5)$$

in dimension $D = 10$ as well as in lower dimensions. It may be written more compactly in terms of the linearized field strengths

$$f_i^{\mu\nu} = p_i^\mu a_i^\nu - p_i^\nu a_i^\mu = -f_i^{\nu\mu} \quad (2.6)$$

thus getting a manifestly gauge invariant expression

$$\begin{aligned} K_{vv \rightarrow vv}^{tree}(a_i, p_i) &= \frac{1}{2} [(f_1 f_2 f_3 f_4) + (f_1 f_3 f_4 f_2) + (f_1 f_4 f_2 f_3)] \\ &- \frac{1}{4} [(f_1 f_2)(f_3 f_4) + (f_1 f_3)(f_4 f_2) + (f_1 f_4)(f_2 f_3)] \end{aligned} \quad (2.7)$$

²We use mostly plus signature for the Lorentz metric $\eta_{\mu\nu} = (-, +, +, +)$.

where

$$(f_i f_j f_k f_l) = f_i^\mu{}_\nu f_j^\nu{}_\rho f_k^\rho{}_\sigma f_l^\sigma{}_\mu \quad (2.8)$$

and

$$(f_i f_j) = f_i^\mu{}_\nu f_j^\nu{}_\mu . \quad (2.9)$$

For forward scattering $p_4 = -p_1$ and $p_3 = -p_2$, so that $t = 0$ and $u = -s$, $a_3 = a_2$ and $a_4 = a_1$, the kinematic factor drastically simplifies to

$$K_{vv \rightarrow vv}^{FS, tree} = (\alpha' s)^2 a_1^2 a_2^2 . \quad (2.10)$$

3. One-loop CP even amplitudes

In this section we compute the worldsheet correlators that appear in the one-loop scattering amplitudes (planar, non-planar and non-orientable) of four open string vector bosons for supersymmetric models with intersecting and magnetized branes on orbifolds³. For the moment we focus on CP even amplitudes that receive contribution from the even spin structure in the NSR formalism. CP odd amplitudes from the odd spin structure are described in the next section.

Up to Chan-Paton factors, the one-loop four vector boson amplitude in the direct channel (‘open string’ description) reads

$$\begin{aligned} \mathcal{A}_{vv \rightarrow vv}^{1-loop}(p_i, a_i) &= g_s^2 \int_0^\infty \frac{dt}{t} \int_{\mathcal{R}} \prod_i dz_i \\ &\sum_\alpha c_\alpha \langle V_0(z_1; p_1, a_1) V_0(z_2; p_2, a_2) V_0(z_3; p_3, a_3) V_0(z_4; p_4, a_4) \rangle_\alpha . \end{aligned} \quad (3.1)$$

The power of the modular parameter t in the denominator takes care of the volume of the conformal Killing group and effectively cancels the integration over the ‘center of mass’ coordinate the correlator is independent of. Additional negative powers of t will appear as a result of integration over loop momentum. Summation over the spin structures α ($\alpha = 2, 3, 4$ even, $\alpha = 1$ odd) with appropriate coefficients c_α implements the GSO projection. Moreover summation over the various kinds of magnetized or intersecting branes, later on labelled by $a = 1, \dots, N_a$ with $N_a = Tr_a(\mathbf{1})$, and the various sectors of the orbifold, later on labelled by $k = 0, \dots, n-1$ for the case of $\Gamma = \mathbf{Z}_n$, is understood.

In the planar case, all the four vectors should belong to the same factor in the Chan-Paton group and the annulus amplitude \mathcal{A} is schematically given by

$$\mathcal{A}_{vv \rightarrow vv}^{plan} = \text{tr}_a(T_1 T_2 T_3 T_4 W_k) \text{tr}_b(W_k) \mathcal{A}_{ab}(1, 2, 3, 4), \quad (3.2)$$

where the discrete Wilson lines W represent the projective embedding of the orbifold group Γ in the Chan-Paton group. The open string vertices are inserted on the same boundary of the worldsheet, $z = -\bar{z}$, and the integration region is given by

$$\mathcal{R}_{\mathcal{A}}^{plan} = \{z_i = iy_i : 0 < y_1 < y_2 < y_3 < y_4 < Im\tau_{\mathcal{A}} = t/2\} . \quad (3.3)$$

³Some properties of elliptic functions and propagators are collected in the appendix that should be consulted for notation and conventions.

Non-planar amplitudes receive contribution also when the vector bosons belong to different factors of the Chan-Paton group, i.e. end on different stacks of D-branes. Up to permutations, depending on the choice of the Chan-Paton matrices for the external legs, the corresponding annulus amplitude \mathcal{A} reads

$$\mathcal{A}_{vv \rightarrow vv}^{nonpl} = \text{tr}_a(W_k T_1 T_2) \text{tr}_b(W_k T_3 T_4) \mathcal{A}_{ab}(1, 2; 3, 4) . \quad (3.4)$$

The integration region is given by is

$$\mathcal{R}_{\mathcal{A}}^{nonpl} = \{z_{1,2} = iy_{1,2}, z_{3,4} = \frac{1}{2} + iy_{3,4} : 0 < y_i < Im\tau_{\mathcal{A}} = t/2\} \quad (3.5)$$

but otherwise unrestricted. This will play a crucial role later on in section 7.

For unoriented strings, the only ones where tadpole cancellation can be achieved thanks to the contribution of the Ω -planes, one has to take into account the contribution of the Möbius strip \mathcal{M} , too. Up to permutations, it reads

$$\mathcal{A}_{vv \rightarrow vv}^{unor} = \text{tr}_a(T_1 T_2 T_3 T_4 W_{2k}^{\Omega}) \mathcal{M}_{a\bar{a}}(1, 2, 3, 4) , \quad (3.6)$$

where W_{2k}^{Ω} implements the action of the worldsheet parity in the Chan-Paton group. The choice of N 's and W 's as well as of the fluxes and intersection angles is tightly constrained by consistency conditions such as RR-tadpole cancellation [77–81]. We will assume that such a choice has been made and shall not discuss this issue any further in this paper. Moreover we will not consider phenomena associated to the presence of anomalous $U(1)$'s discussed in [82–85].

For the Möbius strip \mathcal{M} , the integration region is given by

$$\mathcal{R}_{\mathcal{M}} = \{z_i = iy_i + \frac{1}{2}\delta : 0 < y_1 < y_2 < y_3 < y_4 < Im\tau_{\mathcal{M}} = t/2; \delta = 1, 2\} . \quad (3.7)$$

Given the form of the vector emission vertices there are in principle five different kinds of contributions to the worldsheet correlators:

$$\langle(\partial X)^4\rangle_{\alpha} + \text{“4”} \langle(\partial X)^3(\psi\psi)\rangle_{\alpha} + \text{“6”} \langle(\partial X)^2(\psi\psi)^2\rangle_{\alpha} + \text{“4”} \langle(\partial X)(\psi\psi)^3\rangle_{\alpha} + \langle(\psi\psi)^4\rangle_{\alpha} . \quad (3.8)$$

It is easy to check that the first two kinds of terms vanish for any supersymmetric vacuum configuration after summation over the even spin structures or lack of fermionic zero-modes in the odd spin structure. We are thus left with the last three structures.

Contractions of the spacetime bosonic coordinates, satisfying Neumann boundary conditions, are performed by means of

$$\mathcal{G}_{\Sigma}(z - w) = \frac{1}{2} [\mathcal{G}_{\mathcal{T}}(z - w) + \mathcal{G}_{\mathcal{T}}(z - \tilde{w}) + \mathcal{G}_{\mathcal{T}}(\tilde{z} - w) + \mathcal{G}_{\mathcal{T}}(\tilde{z} - \tilde{w})] , \quad (3.9)$$

where $\tilde{z} = 1 - \bar{z} = z$ and $\tilde{w} = 1 - \bar{w} = w$ for open string insertions on the boundary of $\Sigma = \mathcal{A}, \mathcal{M}$, and $\mathcal{G}_{\mathcal{T}}(z, w)$ is the bosonic propagator (Bargmann kernel) on the covering torus

$$\mathcal{G}_{\mathcal{T}}(z - w) = -\frac{\alpha'}{2} \left[\log \left| \frac{\theta_1(z)}{\theta_1(0)} \right|^2 - \frac{2\pi}{Im\tau} Im(z - w)^2 \right] , \quad (3.10)$$

with $\tau = \tau_\Sigma$.

In the even spin structures, free fermion contractions are performed by means of

$$\mathcal{S}_\alpha(z-w) = \frac{\theta_\alpha(z-w) \theta'_1(0)}{\theta_1(z-w) \theta_\alpha(0)}, \quad (3.11)$$

the fermionic propagator (Szego kernel).

In the odd spin structure, the fermionic propagator may be taken to be

$$\mathcal{S}_1(z-w) = -\partial_z \mathcal{G}(z-w). \quad (3.12)$$

Contractions are weighted by the partition function

$$\mathcal{Z}_\alpha^{\mathcal{N}} = \langle 1 \rangle_{\alpha,k}^{a,b}, \quad (3.13)$$

whose explicit form, as we will momentarily see depends on the sector under consideration, i.e. on the choice of k and a, b that determine the number of preserved supersymmetries \mathcal{N} .

3.1 $\mathcal{N} = 4$ sectors: only CP even amplitudes as in $D = 10$

Actually for $\mathcal{N} = 4$ sectors only the last term in (3.8) contributes, i.e. survives summation over the (even) spin structures. The fermionic contractions give a constant since the lowest derivative, $\alpha' \rightarrow 0$ limit, four-vector amplitude is BPS saturated in these sectors.

$\mathcal{N} = 4$ open string sectors are characterized by $k = 0$ and connect parallel and equally magnetized branes ('neutral' and 'dipole' strings [2, 66]). The partition function is given by

$$\mathcal{Z}_\alpha^{\mathcal{N}=4} = \mathcal{X}_{ab}^{\mathcal{N}=4} \frac{\theta_\alpha^4(0)}{\eta^{12}}, \quad (3.14)$$

where

$$\mathcal{X}_{ab}^{\mathcal{N}=4} = \frac{\int d^4 x_0 \Lambda_{ab}}{2_{GSO} 2\Omega_{orb} (\alpha't)^2} \quad (3.15)$$

takes care of numerical factors and bosonic zero modes. In particular, $\int d^4 x_0 = V_X$ is the (regulated) volume of spacetime, $\int d^4 p_0 \exp(-\pi \alpha' p_0^2) = 1/(\alpha't)^2$, and Λ_{ab} denotes the 6-dimensional sum over generalized KK momenta. The numerical factors result from the various projections \mathbf{Z}_2^{GSO} , \mathbf{Z}_2^Ω and $\mathbf{Z}_n^{orbifold}$.

Up to permutations, the (non)planar annulus amplitude reads

$$\begin{aligned} \mathcal{A}_{ab}(1, 2, 3, 4)(p_i, a_i) &= g_s^2 \frac{(2\pi)^4}{4n} \delta\left(\sum_i p_i\right) K_{vv \rightarrow vv}^{tree}(p_i; a_i) \times \\ &\int_0^\infty \frac{dt}{t^3} \Lambda_{ab}(\tau_A) \int_{\mathcal{R}_A^{(non)plan}} \prod_k dz_k \prod_{i < j} e^{-p_i \cdot p_j \mathcal{G}_A(z_{ij})}, \end{aligned} \quad (3.16)$$

where \mathcal{G}_A is the free bosonic propagator on the boundary of the annulus \mathcal{A} (3.9). The integration regions \mathcal{R}_A^{plan} and $\mathcal{R}_A^{non-plan}$ have been discussed above. As indicated, the kinematic factor $K_{vv \rightarrow vv}^{tree}$ is exactly the same as at tree level.

The unoriented Möbius amplitude reads

$$\begin{aligned} \mathcal{M}_{a\bar{a}}(1, 2, 3, 4) &= g_s^2 \frac{(2\pi)^4}{4n} \delta\left(\sum_i p_i\right) K_{vv \rightarrow vv}^{tree}(p_i; a_i) \times \\ &\int_0^\infty \frac{dt}{t^3} \Lambda_{a\bar{a}}(\tau_{\mathcal{A}}) \int_{\tilde{\mathcal{R}}_{\mathcal{M}}} \prod_k dz_k \prod_{i < j} e^{-p_i \cdot p_j \mathcal{G}_{\mathcal{M}}(z_{ij})}, \end{aligned} \quad (3.17)$$

where $\mathcal{G}_{\mathcal{M}}$ is the free boson propagator on the boundary of the Möbius strip (3.9).

3.2 $\mathcal{N} = 1, 2$ sectors: CP even amplitudes

Let us consider the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetric sectors that can be analyzed in parallel in the even spin structures. The odd spin structure needs a separate analysis. The main difference between the two cases resides in the internal contribution. Our analyses applies to arbitrary choices (‘parallel’ [2] or ‘oblique’ [66]) of constant abelian magnetic fluxes⁴ or intersecting angles in (supersymmetric) orbifold compactifications.

In the $\mathcal{N} = 1$ case one has

$$\langle 1 \rangle_{\alpha, k, \varepsilon}^{a, b} = \mathcal{Z}_{\alpha}^{\mathcal{N}=1}(u_{ab}^I) = \mathcal{X}_{ab}^{\mathcal{N}=1} \frac{\theta_{\alpha}(0)}{\eta^3} \prod_I \frac{\theta_{\alpha}(u_{ab}^I)}{\theta_1(u_{ab}^I)}, \quad (3.18)$$

where

$$\mathcal{X}_{ab}^{\mathcal{N}=1} = \frac{\mathcal{I}_{ab} \int d^4 x_0}{2_{GSO} 2_{\Omega} n_{orb} (\alpha' t)^2} \quad (3.19)$$

\mathcal{I}_{ab} is some discrete multiplicity, e.g. degeneracy of Landau levels, number of fixed points or intersections,

$$u_{ab}^I = \varepsilon_{ab}^I \tau_{\mathcal{A}} + k v_{ab}^I, \quad (3.20)$$

with ε_{ab}^I denoting intersection angles or magnetic shifts and $k v_{ab}^I$ implementing some \mathbf{Z}_n orbifold projection with $k = 0, 1, \dots, n - 1$. $\mathcal{N} = 1$ supersymmetry requires $u_{ab}^I \neq 0$, with $\sum_I u_{ab}^I = 0 \pmod{1}$.

In the $\mathcal{N} = 2$ case one of the u_{ab}^I vanishes. For definiteness let us set $u_{ab}^3 = 0$, then $u_{ab}^1 = -u_{ab}^2 = u_{ab} \pmod{1}$ and one obtains

$$\langle 1 \rangle_{\alpha, k, \varepsilon}^{a, b} = \mathcal{Z}_{\alpha}^{\mathcal{N}=2}(u_{ab}) = \mathcal{X}_{ab}^{\mathcal{N}=2} \frac{\theta_{\alpha}(0)^2 \theta_{\alpha}(u_{ab}) \theta_{\alpha}(-u_{ab})}{\eta^6 \theta_1(u_{ab}) \theta_1(-u_{ab})}, \quad (3.21)$$

where

$$\mathcal{X}_{ab}^{\mathcal{N}=2} = \frac{\int d^4 x_0 \mathcal{I}_{ab}^{\perp} \Lambda_{ab}^{\parallel}(\tau_{\mathcal{A}})}{2_{GSO} 2_{\Omega} n_{orb} (\alpha' t)^2}. \quad (3.22)$$

In addition to the discrete multiplicity \mathcal{I}_{ab}^{\perp} in the twisted or magnetized directions, a sum of generalized KK momenta $\Lambda_{ab}^{\parallel}(\tau_{\mathcal{A}})$ in the untwisted or unmagnetized directions is present.

In both cases the Chan-Paton factors get modified to $\text{tr}_a(T^1 \dots W^k)$ by the effect of (discrete) Wilson lines W corresponding to the projective embedding of the orbifold group Γ in the gauge group. Notice that the unbroken gauge group G_a^u for branes of type a corresponds to the generators T_a^u such that $[T_a^u, W_a^k] = 0$.

Let us now consider the Wick contractions one at a time.

⁴A worldsheet description of the non abelian fluxes discussed in [86] is not yet available.

3.2.1 Two fermion bilinears (6 terms)

Up to permutations (six in all) the typical $\langle (\partial X)^2 (\psi\psi)^2 \rangle_\alpha$ correlator reads

$$\langle a_1 \cdot \partial X e^{ip_1 \cdot X}(z_1) a_2 \cdot \partial X e^{ip_2 \cdot X}(z_2) e^{ip_3 \cdot X}(z_3) e^{ip_4 \cdot X}(z_4) \rangle \langle ip_3 \cdot \psi a_3 \cdot \psi(z_3) ip_4 \cdot \psi a_4 \cdot \psi(z_4) \rangle_\alpha. \quad (3.23)$$

The bosonic correlator yields

$$\begin{aligned} & \langle a_1 \cdot \partial X e^{ip_1 \cdot X}(z_1) a_2 \cdot \partial X e^{ip_2 \cdot X}(z_2) e^{ip_3 \cdot X}(z_3) e^{ip_4 \cdot X}(z_4) \rangle = \\ & [a_1 \cdot a_2 \partial_1 \partial_2 \mathcal{G}(z_{12}) - \sum_{i \neq 1} a_1 \cdot p_i \partial_1 \mathcal{G}(z_{1i}) \sum_{j \neq 2} a_2 \cdot p_j \partial_2 \mathcal{G}(z_{2j})] \Pi(p_i, z_i), \end{aligned} \quad (3.24)$$

where \mathcal{G} is the free bosonic propagator defined in (3.9) and

$$\Pi(p_i, z_i) = \prod_{i < j} \exp[-p_i \cdot p_j \mathcal{G}(z_{ij})]. \quad (3.25)$$

The fermionic correlator yields

$$\langle ip_3 \cdot \psi a_3 \cdot \psi(z_3) ip_4 \cdot \psi a_4 \cdot \psi(z_4) \rangle_\alpha = -2 \frac{1}{2^2} (f_3 f_4) \mathcal{S}_\alpha^2(z_{34}) \mathcal{Z}_\alpha, \quad (3.26)$$

where \mathcal{S}_α is the fermionic propagator (Szegő kernel) for even spin structures defined in (3.11). Using

$$\mathcal{S}_\alpha^2(z-w) = \mathcal{P}(z-w) - e_{\alpha-1}, \quad (3.27)$$

where $\mathcal{P}(z)$ is Weierstrass \mathcal{P} function

$$\mathcal{P}(z) = -\partial_z^2 \log \theta_1(z) - 2\eta_1, \quad (3.28)$$

with

$$\eta_1 = -2\pi i \partial_\tau \log \eta = -\frac{1}{6} \frac{\theta_1'''(0)}{\theta_1'(0)}, \quad (3.29)$$

since $\theta_1'(0) = 2\pi\eta^3$, and

$$e_{\alpha-1} = -4\pi i \frac{d}{d\tau} \log \frac{\theta_\alpha(0|\tau)}{\eta(\tau)}, \quad (3.30)$$

it is easy to see that only the term $\partial_\tau \log \theta_\alpha$ in $e_{\alpha-1}$ survives summation over the even spin structures.

For $\mathcal{N} = 1$ sectors, one finds

$$\mathcal{E}_{\mathcal{N}=1}(u_{ab}^I) = \mathcal{X}_{ab}^{\mathcal{N}=1} \sum_\alpha \frac{\theta_\alpha''(0)}{\eta^3} \prod_I \frac{\theta_\alpha(u_{ab}^I)}{\theta_1(u_{ab}^I)} = 2\pi \mathcal{X}_{ab}^{\mathcal{N}=1} \sum_I \frac{\theta_1'(u_{ab}^I)}{\theta_1(u_{ab}^I)} = 2\pi \mathcal{X}_{ab}^{\mathcal{N}=1} \frac{\mathcal{H}'(0)}{\mathcal{H}(0)}, \quad (3.31)$$

where

$$\mathcal{H}(z) = \prod_I \theta_1(z + u_{ab}^I) \quad (3.32)$$

and the zero-mode factor $\mathcal{X}_{ab}^{\mathcal{N}=1}$ is defined in (3.19).

For $\mathcal{N} = 2$ sectors one finds

$$\mathcal{E}_{\mathcal{N}=2}(u_{ab}) = \mathcal{X}_{ab}^{\mathcal{N}=2} \sum_{\alpha} \frac{\theta_{\alpha}''(0)\theta_{\alpha}(0)\theta_{\alpha}(u_{ab})\theta_{\alpha}(-u_{ab})}{\eta^6\theta_1(u_{ab})\theta_1(-u_{ab})} = 4\pi^2 \mathcal{X}_{ab}^{\mathcal{N}=2}, \quad (3.33)$$

where the zero-mode factor $\mathcal{X}_{ab}^{\mathcal{N}=2}$ is defined in (3.22).

Thus, eventually the fermionic correlator

$$\langle ip_3 \cdot \psi_{a_3} \cdot \psi(z_3) ip_4 \cdot \psi_{a_4} \cdot \psi(z_4) \rangle_{even} = -\frac{1}{2}(f_3 f_4) \mathcal{E}_{\mathcal{N}=1,2}(u_{ab}^I) \quad (3.34)$$

turns out to be independent of the insertion points in $\mathcal{N} = 1, 2$ sectors. As already stated, it receives vanishing contribution from $\mathcal{N} = 4$ sectors.

3.2.2 Three fermion bilinears (4 terms)

Up to permutations (four in all) the correlator $\langle (\psi\psi)^3(\partial X) \rangle_{\alpha}$ can be most conveniently computed by observing that normal ordering of the fermion bilinears $:\psi\psi:$ allows only for a cyclic Lorentz contraction that yields

$$\langle ip_1 \cdot \psi_{a_1} \cdot \psi ip_2 \cdot \psi_{a_2} \cdot \psi ip_3 \cdot \psi_{a_3} \cdot \psi \rangle_{\alpha} = -i \frac{2^3}{3} (f_1 f_2 f_3) \mathcal{S}_{\alpha}(z_{12}) \mathcal{S}_{\alpha}(z_{23}) \mathcal{S}_{\alpha}(z_{13}) \mathcal{Z}_{\alpha}^{\mathcal{N}}, \quad (3.35)$$

where $z_{ij} = z_i - z_j$ and

$$(f_1 f_2 f_3) = f_1^{\mu} f_2^{\nu} f_3^{\rho} f_{3\mu}^{\rho}. \quad (3.36)$$

Using the identity

$$\mathcal{S}_{\alpha}(z_{13}) \mathcal{S}_{\alpha}(z_{23}) = \mathcal{S}_{\alpha}(z_{12}) \omega(z_1, z_2, z_3) + \mathcal{S}'_{\alpha}(z_{12}), \quad (3.37)$$

where

$$\omega(z_1, z_2, z_3) = \partial_1 \log \theta_1(z_{12}) + \partial_2 \log \theta_1(z_{23}) + \partial_3 \log \theta_1(z_{31}), \quad (3.38)$$

re-combining the two $\mathcal{S}_{\alpha}(z_{12})$, using (3.27) and

$$\mathcal{S}_{\alpha}(z) \mathcal{S}'_{\alpha}(z) = \frac{1}{2} \partial_z (\mathcal{P}(z) - e_{\alpha-1}) = \frac{1}{2} \mathcal{P}'(z), \quad (3.39)$$

and summing over spin structures yields

$$\langle ip_1 \cdot \psi_{a_1} \cdot \psi ip_2 \cdot \psi_{a_2} \cdot \psi ip_3 \cdot \psi_{a_3} \cdot \psi \rangle_{even} = -i (f_1 f_2 f_3) \omega(z_1, z_2, z_3) \mathcal{E}_{\mathcal{N}}(u_{ab}^I), \quad (3.40)$$

which is manifestly symmetric under any permutation of the three insertion points. As already stated, this correlator gets no contribution from $\mathcal{N} = 4$ sectors.

The bosonic correlator simply yields

$$\langle e^{ip_1 \cdot X} e^{ip_2 \cdot X} e^{ip_3 \cdot X} a_4 \cdot \partial X e^{ip_4 \cdot X} \rangle = i \sum_{i \neq 4} a_4 \cdot p_i \partial_4 \mathcal{G}(z_{i4}) \Pi(p_i, z_i), \quad (3.41)$$

where \mathcal{G} is the bosonic propagator 3.9 and $\Pi(p_i, z_i)$ is the momentum factor (3.25).

3.2.3 Four fermion bilinears (1 term, 2 structures)

Let us finally consider the $\langle(\psi\psi)^4\rangle_\alpha$ term. The bosonic coordinates contribute the momentum factor $\Pi(z_i; p_i)$ defined in (3.25). Taking into account normal ordering of the $:\psi\psi:$'s allows for two kinds of contraction.

Three are connected contractions of the Lorentz indices that yield

$$\langle(\psi\psi)^4\rangle_\alpha^{conn} = \frac{2^4}{2^4}(f_1 f_2 f_3 f_4) \mathcal{S}_\alpha(z_{12}) \mathcal{S}_\alpha(z_{23}) \mathcal{S}_\alpha(z_{34}) \mathcal{S}_\alpha(z_{14}) \mathcal{Z}_\alpha, \quad (3.42)$$

where

$$(f_1 f_2 f_3 f_4) = f_1^\mu{}_\nu f_2^\nu{}_\rho f_3^\rho{}_\lambda f_4^\lambda{}_\mu. \quad (3.43)$$

Using (3.37), the product of fermionic propagators can be simplified to

$$\begin{aligned} & [\mathcal{S}_\alpha(z_{13})\omega(z_1, z_2, z_3) + \mathcal{S}'_\alpha(z_{13})][\mathcal{S}_\alpha(z_{13})\omega(z_1, z_4, z_3) + \mathcal{S}'_\alpha(z_{13})] \\ = & \omega(z_1, z_2, z_3)\omega(z_1, z_4, z_3)[\mathcal{P}(z_{13}) - e_{\alpha-1}] \\ & + \frac{1}{2}[\omega(z_1, z_2, z_3) + \omega(z_1, z_4, z_3)]\mathcal{P}'(z_{13}) + \mathcal{S}'_\alpha(z_{13})^2. \end{aligned} \quad (3.44)$$

Summing over spin structures only the first and the last term survive. Let us denote them by $\mathcal{U}_\mathcal{N}(z_i)$ and $\mathcal{V}_\mathcal{N}(z_i)$. The former simply reads

$$\mathcal{U}_\mathcal{N}(z_i) = -\omega(z_1, z_2, z_3)\omega(z_1, z_4, z_3)\mathcal{E}_\mathcal{N}(u_{ab}^I). \quad (3.45)$$

The latter is more laborious. Observing that

$$\mathcal{S}'_\alpha(z)^2 = \partial_z[\mathcal{S}_\alpha(z)\mathcal{S}'_\alpha(z)] - \mathcal{S}_\alpha(z)\mathcal{S}''_\alpha(z), \quad (3.46)$$

which, using (3.27), in turn gives

$$\mathcal{S}'_\alpha(z)^2 = \frac{1}{2}\mathcal{P}''(z) - \mathcal{S}_\alpha(z)\mathcal{S}''_\alpha(z) \quad (3.47)$$

it is clear that only the second term contributes after summation over the spin structures, so that

$$\mathcal{V}_\mathcal{N}(z_i) = \lim_{z_0 \rightarrow z_1} \partial_{z_0}^2 \sum_\alpha c_\alpha \mathcal{S}_\alpha(z_{03}) \mathcal{S}_\alpha(z_{13}) \mathcal{Z}_\alpha^\mathcal{N}. \quad (3.48)$$

Further using (3.37) in the following guise

$$\mathcal{S}_\alpha(z_{03})\mathcal{S}_\alpha(z_{13}) = \mathcal{S}_\alpha(z_{01}) \left[\omega_{z_0-z_1}(z_3) + \frac{\theta'_\alpha(z_{01})}{\theta_\alpha(z_{01})} \right], \quad (3.49)$$

where

$$\omega_{x-y}(z) = \partial_z \log \frac{\theta_1(z-x)}{\theta_1(z-y)} \quad (3.50)$$

is a differential of the third kind with two simple poles with opposite residues (± 1) at $z = x$ and $z = y$, yields

$$\mathcal{V}_\mathcal{N}(z_i) = - \lim_{z_0 \rightarrow z_1} \partial_{z_0}^2 \sum_\alpha c_\alpha - \left[\frac{\theta_\alpha(z_{01})}{\theta_1(z_{01})} \omega_{z_0-z_1}(z_3) + \frac{\theta_\alpha(z_{01})'}{\theta_1(z_{01})} \right] \mathcal{Z}_\alpha^\mathcal{N}. \quad (3.51)$$

Both terms can be computed by means of the Riemann identity for even spin structures

$$\sum_{\alpha} c_{\alpha} \theta_{\alpha}(z_1) \theta_{\alpha}(z_2) \theta_{\alpha}(z_3) \theta_{\alpha}(z_4) = \theta_1(z'_1) \theta_1(z'_2) \theta_1(z'_3) \theta_1(z'_4) - \theta_1(z''_1) \theta_1(z''_2) \theta_1(z''_3) \theta_1(z''_4), \quad (3.52)$$

where z'_i and z''_i are related to z_i through

$$\begin{aligned} z'_1 &= \frac{1}{2}(z_1 + z_2 + z_3 + z_4) & z'_2 &= \frac{1}{2}(z_1 + z_2 - z_3 - z_4) \\ z'_3 &= \frac{1}{2}(z_1 - z_2 + z_3 - z_4) & z'_4 &= \frac{1}{2}(z_1 - z_2 - z_3 + z_4) \end{aligned} \quad (3.53)$$

and

$$\begin{aligned} z''_1 &= \frac{1}{2}(-z_1 + z_2 + z_3 + z_4) & z''_2 &= \frac{1}{2}(z_1 - z_2 + z_3 + z_4) \\ z''_3 &= \frac{1}{2}(z_1 + z_2 - z_3 + z_4) & z''_4 &= \frac{1}{2}(z_1 + z_2 + z_3 - z_4). \end{aligned} \quad (3.54)$$

For $\mathcal{N} = 1$ sectors

$$\begin{aligned} \mathcal{V}_{\mathcal{N}=1}(z_i) &= -2\pi \mathcal{X}_{ab}^{\mathcal{N}=1} \times \\ \lim_{z_0 \rightarrow z_1} \partial_{z_0}^2 &\left[\frac{(\omega_{z_0-z_1}(z_3) + \partial_{z_0}) \{ \theta_1(z_{01}/2) [\mathcal{H}(z_{01}/2) - \mathcal{H}(-z_{01}/2)] \}}{\theta_1(z_{01}) \mathcal{H}(0)} \right], \end{aligned} \quad (3.55)$$

where $\mathcal{H}(z)$ is defined in (3.32), that eventually yields

$$\begin{aligned} \mathcal{V}_{\mathcal{N}=1}(z_i) &= -2\pi \mathcal{X}_{ab}^{\mathcal{N}=1} \frac{\mathcal{H}'(0)}{\mathcal{H}(0)} \left[\partial_3 \frac{\theta'_1(z_{31})}{\theta_1(z_{31})} + \frac{1}{6} \frac{\theta'''_1(0)}{\theta'_1(0)} - \frac{1}{6} \frac{\mathcal{H}'''(0)}{\mathcal{H}'(0)} \right] \\ &= \mathcal{E}_{\mathcal{N}=1} \mathcal{P}(z_{13}) + \mathcal{F}_{\mathcal{N}=1}, \end{aligned} \quad (3.56)$$

where

$$\mathcal{F}_{\mathcal{N}=1} = 2\pi \mathcal{X}_{ab}^{\mathcal{N}=1} \frac{\mathcal{H}'(0)}{\mathcal{H}(0)} \left[3\eta_1 + \frac{1}{6} \frac{\mathcal{H}'''(0)}{\mathcal{H}'(0)} \right]. \quad (3.57)$$

Including the bosonic contraction producing the momentum factor (3.25) and adding the term (3.45), one eventually finds

$$\begin{aligned} \langle (\psi\psi)^4 \rangle_{conn}^{\mathcal{N}=1} &= \frac{1}{2} (f_1 f_2 f_3 f_4) \Pi(p_i, z_i) \{ 2\mathcal{F}_{\mathcal{N}=1} + \\ &+ \mathcal{E}_{\mathcal{N}=1} [\mathcal{P}(z_{13}) - \omega(z_1, z_2, z_3) \omega(z_1, z_4, z_3) + \mathcal{P}(z_{24}) - \omega(z_2, z_1, z_4) \omega(z_2, z_3, z_4)] \}, \end{aligned} \quad (3.58)$$

where $\mathcal{E}_{\mathcal{N}=1}$ has been defined above (3.31). Symmetry under the exchange of z_1, z_3 with z_2, z_4 has been made explicit though unnecessary, since we expect it to be

$$\mathcal{P}(z_{31}) - \omega(z_1, z_2, z_3) \omega(z_1, z_4, z_3) = \mathcal{P}(z_{24}) - \omega(z_2, z_1, z_4) \omega(z_2, z_3, z_4) \quad (3.59)$$

from consideration of periodicity and singularity.

For $\mathcal{N} = 2$ sectors, ones finds

$$\mathcal{V}_{\mathcal{N}=2}(z_i) = -4\pi^2 \mathcal{X}_{ab}^{\mathcal{N}=2} \lim_{z_0 \rightarrow z_1} \partial_{z_0}^2 \sum_{\alpha} c_{\alpha} \frac{\theta_{\alpha}(z_{03})}{\theta_1(z_{03})} \frac{\theta_{\alpha}(z_{13})}{\theta_1(z_{13})} \frac{\theta_{\alpha}(u_{ab})}{\theta_1(u_{ab})} \frac{\theta_{\alpha}(-u_{ab})}{\theta_1(-u_{ab})}. \quad (3.60)$$

The limit yields

$$\begin{aligned} & \partial_{z_0}^2 \left[\theta_1 \left(\frac{z_{03} + z_{13}}{2} \right) \theta_1 \left(\frac{z_{03} + z_{13}}{2} \right) \theta_1 \left(\frac{z_{03} - z_{13}}{2} - u_{ab} \right) \theta_1 \left(\frac{z_{03} - z_{13}}{2} + u_{ab} \right) + \right. \\ & \left. \theta_1 \left(\frac{z_{03} - z_{13}}{2} \right) \theta_1 \left(\frac{z_{03} - z_{13}}{2} \right) \theta_1 \left(\frac{z_{03} + z_{13}}{2} - u_{ab} \right) \theta_1 \left(\frac{z_{03} + z_{13}}{2} + u_{ab} \right) \right]_{z_0=z_1} \\ & = [\mathcal{P}(z_{31}) - \mathcal{P}(u_{ab})] \theta_1(z_{13})^2 \theta_1(-u_{ab})^2 . \end{aligned} \quad (3.61)$$

So that, including the momentum factor (3.25), one eventually obtains

$$\begin{aligned} \langle (\psi\psi)^4 \rangle_{conn}^{\mathcal{N}=2} &= \frac{1}{2} (f_1 f_2 f_3 f_4) \Pi(p_i, z_i) \{ 2\mathcal{F}_{\mathcal{N}=2} - \\ & + \mathcal{E}_{\mathcal{N}=2} [\mathcal{P}(z_{13}) - \omega(z_1, z_2, z_3) \omega(z_1, z_4, z_3) + \mathcal{P}(z_{24}) - \omega(z_2, z_1, z_4) \omega(z_2, z_3, z_4)] \} , \end{aligned} \quad (3.62)$$

where

$$\mathcal{F}_{\mathcal{N}=2} = -\mathcal{E}_{\mathcal{N}=2} \mathcal{P}(u_{ab}) = -4\pi^2 \mathcal{X}_{ab}^{\mathcal{N}=2} \mathcal{P}(u_{ab}) . \quad (3.63)$$

The other kind of disconnected contractions lead to three inequivalent possibilities that yield terms of the form

$$\langle (\psi\psi)^4 \rangle_{\alpha}^{disc} = \frac{2^2}{2^4} (f_1 f_2) (f_3 f_4) \mathcal{S}_{\alpha}^2(z_{12}) \mathcal{S}_{\alpha}^2(z_{34}) \mathcal{Z}_{\alpha}^{\mathcal{N}} . \quad (3.64)$$

Dropping the kinematical factor, summation over spin structures yields

$$[\mathcal{P}(z_{12}) + \mathcal{P}(z_{34})] \mathcal{E}_{\mathcal{N}} + \mathcal{F}_{\mathcal{N}} , \quad (3.65)$$

where

$$\mathcal{F}_{\mathcal{N}} = \sum_{\alpha} c_{\alpha} e_{\alpha-1}^2 \mathcal{Z}_{\alpha}^{\mathcal{N}} \quad (3.66)$$

turns out to hold for the $\mathcal{F}_{\mathcal{N}}$ previously defined in (3.57) and (3.63). Indeed, (3.66) can be simplified using (3.37) and [76]

$$e_{\alpha-1}^2 = i\pi \partial_{\tau} e_{\alpha-1} + 2\eta_1 e_{\alpha-1} + \frac{1}{6} g_2 \quad (3.67)$$

to

$$\mathcal{F}_{\mathcal{N}} = 2\eta_1 \mathcal{E}_{\mathcal{N}} - \frac{1}{4} \sum_{\alpha} c_{\alpha} \left[\frac{\theta_{\alpha}^{(4)}(0)}{\theta_{\alpha}(0)} - \left(\frac{\theta_{\alpha}''(0)}{\theta_{\alpha}(0)} \right)^2 \right] \mathcal{Z}_{\alpha}^{\mathcal{N}} . \quad (3.68)$$

In $\mathcal{N} = 1$ sectors one finds

$$\sum_{\alpha} c_{\alpha} \frac{\theta_{\alpha}^{(4)}(0)}{\eta^3} \prod_I \frac{\theta_{\alpha}(u^I)}{\theta_1(u^I)} = \frac{1}{2} \mathcal{E}_{\mathcal{N}=1} \left[\frac{\mathcal{H}'''}{\mathcal{H}'} - 6\eta_1 \right] \quad (3.69)$$

and

$$\sum_{\alpha} c_{\alpha} \frac{\theta_{\alpha}''(0)^2}{\eta^3 \theta_{\alpha}(0)} \prod_I \frac{\theta_{\alpha}(u^I)}{\theta_1(u^I)} = \frac{1}{6} \sum_{\alpha} c_{\alpha} \frac{\partial_z^4 [\theta_{\alpha}(z)^2]_{z=0} - 2\theta_{\alpha}(0) \theta_{\alpha}^{(4)}(0)}{\eta^3 \theta_{\alpha}(0)} \prod_I \frac{\theta_{\alpha}(u^I)}{\theta_1(u^I)} . \quad (3.70)$$

The second term is given in (3.69), while the first can be computed by means of the identity

$$\frac{\theta_\alpha(z)^2}{\theta_\alpha(0)} = \frac{\mathcal{S}_\alpha(z)^2 \theta_1(z)^2 \theta_\alpha(0)}{\theta'_1(0)^2} = \frac{[\mathcal{P}(z) - e_{\alpha-1}] \theta_1(z)^2 \theta_\alpha(0)}{\theta'_1(0)^2}, \quad (3.71)$$

that after differentiation and summation over the spin structures yields

$$\frac{1}{6} 2\pi \mathcal{X}_{ab}^{\mathcal{N}=1} \sum_\alpha c_\alpha \frac{\partial_z^4 [\theta_\alpha(z)^2]_{z=0}}{\eta^3 \theta_\alpha(0)} \prod_I \frac{\theta_\alpha(u_{ab}^I)}{\theta_1(u_{ab}^I)} = -8\eta_1 \mathcal{E}_{\mathcal{N}=1} \quad (3.72)$$

so that eventually one indeed finds

$$\mathcal{F}_{\mathcal{N}=1} = \mathcal{E}_{\mathcal{N}=1} \left[\frac{1}{6} \frac{\mathcal{H}'''(0)}{\mathcal{H}'(0)} + 3\eta_1 \right]. \quad (3.73)$$

as above (3.57). Including the momentum factor (3.25) yields

$$\langle (\psi\psi)^4 \rangle_{disc}^{\mathcal{N}=1} = \frac{1}{4} (f_1 f_2) (f_3 f_4) \{ \mathcal{E}_{\mathcal{N}=1} [\mathcal{P}(z_{12}) + \mathcal{P}(z_{34})] + \mathcal{F}_{\mathcal{N}=1} \} \Pi(p_i, z_i). \quad (3.74)$$

In $\mathcal{N} = 2$ sectors one has

$$\begin{aligned} \mathcal{F}_{\mathcal{N}=2} &= -\frac{\mathcal{X}^{\mathcal{N}=2}}{\eta^6} \sum_\alpha c_\alpha [\theta'_\alpha(0) - 2\eta_1 \theta_\alpha(0)]^2 \frac{\theta_\alpha(u_{ab})^2}{\theta_1(u_{ab})^2} \\ &= 4\eta_1 \mathcal{E}_{\mathcal{N}=2} - \frac{\mathcal{X}_{ab}^{\mathcal{N}=2}}{\eta^6} \sum_\alpha c_\alpha \theta''_\alpha(0)^2 \frac{\theta_\alpha(u_{ab})^2}{\theta_1(u_{ab})^2}. \end{aligned} \quad (3.75)$$

The last sum leads to

$$\begin{aligned} & -\partial_z^2 \partial_w^2 \left[\theta_1 \left(\frac{z+w}{2} \right) \theta_1 \left(\frac{z+w}{2} \right) \theta_1 \left(\frac{z-w}{2} - u_{ab} \right) \theta_1 \left(\frac{z-w}{2} + u_{ab} \right) + \right. \\ & \left. \theta_1 \left(\frac{z-w}{2} \right) \theta_1 \left(\frac{z-w}{2} \right) \theta_1 \left(\frac{z+w}{2} - u_{ab} \right) \theta_1 \left(\frac{z+w}{2} + u_{ab} \right) \right]_{z=w=0} \\ &= -\theta'_1(0)^2 [\partial_u^2 \log \theta_1(u_{ab}) - 2\eta_1] = \theta'_1(0)^2 \mathcal{P}(u_{ab}). \end{aligned} \quad (3.76)$$

Including the momentum factor finally yields

$$\langle (\psi\psi)^4 \rangle_{disc}^{\mathcal{N}=2} = \frac{1}{4} (f_1 f_2) (f_3 f_4) \{ \mathcal{E}_{\mathcal{N}=2} [\mathcal{P}(z_{12}) + \mathcal{P}(z_{34})] - \mathcal{F}_{\mathcal{N}=2} \} \Pi(p_i, z_i), \quad (3.77)$$

where

$$\mathcal{F}_{\mathcal{N}=2} = -\mathcal{E}_{\mathcal{N}=2} \mathcal{P}(u_{ab}) \quad (3.78)$$

as above (3.63) and $\mathcal{E}_{\mathcal{N}=2}$ is defined in (3.33).

4. Summary of the results for CP even amplitudes

Let us summarize our results in the NSR formalism according to the number of supersymmetries preserved for the CP even amplitudes receiving contribution from the sum over even spin structures.

4.1 No fermion bilinears

$$\langle a_1 \cdot \partial X e^{ip_1 \cdot X}(z_1) a_2 \cdot \partial X e^{ip_2 \cdot X}(z_2) a_3 \cdot \partial X e^{ip_3 \cdot X}(z_3) a_4 \cdot \partial X e^{ip_4 \cdot X}(z_4) \rangle_{\text{even}} = 0 \quad (4.1)$$

in any supersymmetric sector after summing over the even spin structures.

4.2 One fermion bilinear

$$\langle f_{\mu\nu}^1 \psi^\mu \psi^\nu e^{ip_1 \cdot X}(z_1) a_2 \cdot \partial X e^{ip_2 \cdot X}(z_2) a_3 \cdot \partial X e^{ip_3 \cdot X}(z_3) a_4 \cdot \partial X e^{ip_4 \cdot X}(z_4) \rangle_{\text{even}} = 0 \quad (4.2)$$

due to normal ordering of the fermion bilinear.

4.3 Two fermion bilinears

$$\begin{aligned} & \langle \frac{i}{2} f_{\mu_1 \nu_1}^1 \psi^{\mu_1} \psi^{\nu_1} e^{ip_1 \cdot X}(z_1) \frac{i}{2} f_{\mu_2 \nu_2}^2 \psi^{\mu_2} \psi^{\nu_2} e^{ip_2 \cdot X}(z_2) a_3 \cdot \partial X e^{ip_3 \cdot X}(z_3) a_4 \cdot \partial X e^{ip_4 \cdot X}(z_4) \rangle_{\text{even}} \\ &= -\frac{1}{2} (f_1 f_2) \mathcal{E}_{\mathcal{N}} \Pi(z_i; p_i) \left[a_3 \cdot a_4 \partial_3 \partial_4 \mathcal{G}_{34} - \sum_{i \neq 3} a_3 \cdot p_i \partial_3 \mathcal{G}_{3i} \sum_{j \neq 4} a_4 \cdot p_j \partial_4 \mathcal{G}_{4j} \right] \end{aligned}$$

plus permutations (6 in all), where

$$\Pi(p_i) = \prod_{i < j} \exp(-p_i \cdot p_j \mathcal{G}_{ij}) \quad (4.3)$$

and, depending on the number of supersymmetries \mathcal{N} ,

$$\mathcal{E}_{\mathcal{N}=4} = 0 \quad , \quad \mathcal{E}_{\mathcal{N}=2} = (2\pi)^2 \mathcal{X}_{ab}^{\mathcal{N}=2} \quad , \quad \mathcal{E}_{\mathcal{N}=1} = 2\pi \mathcal{X}_{ab}^{\mathcal{N}=1} \frac{\mathcal{H}'(0)}{\mathcal{H}(0)} \quad , \quad (4.4)$$

with $\mathcal{H}(z) = \prod_I \theta_1(z + u_{ab}^I)$ and, up to $\delta(\sum_i p_i)$,

$$\mathcal{X}_{ab}^{\mathcal{N}=4} = \frac{(2\pi)^4 \Lambda_{ab}}{4n(\alpha' t)^2} \quad , \quad \mathcal{X}_{ab}^{\mathcal{N}=2} = \frac{(2\pi)^4 \mathcal{I}_{ab}^\perp \Lambda_{ab}^\parallel}{4n(\alpha' t)^2} \quad , \quad \mathcal{X}_{ab}^{\mathcal{N}=1} = \frac{(2\pi)^4 \mathcal{I}_{ab}}{4n(\alpha' t)^2} \quad . \quad (4.5)$$

4.4 Three fermion bilinears

$$\begin{aligned} & \langle \frac{i}{2} f_{\mu_1 \nu_1}^1 \psi^{\mu_1} \psi^{\nu_1} e^{ip_1 \cdot X}(z_1) \dots \frac{i}{2} f_{\mu_3 \nu_3}^3 \psi^{\mu_3} \psi^{\nu_3} e^{ip_3 \cdot X}(z_3) a_4 \cdot \partial X e^{ip_4 \cdot X}(z_4) \rangle_{\text{even}} \\ &= (f_1 f_2 f_3) \mathcal{E}_{\mathcal{N}} \omega(z_1, z_2, z_3) \sum_{j \neq 4} a_4 \cdot p_j \partial_4 \mathcal{G}_{4j} \Pi(z_i; p_i) \end{aligned}$$

plus permutations (4 in all) where

$$\omega(z_1, z_2, z_3) = \partial_1 \log \theta_1(z_{12}) + \partial_2 \log \theta_1(z_{23}) + \partial_3 \log \theta_1(z_{31}) = \partial_1 \mathcal{G}_{12} + \partial_2 \mathcal{G}_{23} + \partial_3 \mathcal{G}_{31} \quad . \quad (4.6)$$

4.5 Four fermion bilinears, connected

$$\begin{aligned} & \langle \frac{i}{2} f_{\mu_1 \nu_1}^1 \psi^{\mu_1} \psi^{\nu_1} e^{ip_1 \cdot X}(z_1) \dots \frac{i}{2} f_{\mu_4 \nu_4}^4 \psi^{\mu_4} \psi^{\nu_4} e^{ip_4 \cdot X}(z_4) \rangle_{\text{even}}^{\text{conn}} = \frac{1}{2} (f_1 f_2 f_3 f_4) \times \\ & \Pi(z_i; p_i) [\mathcal{E}_{\mathcal{N}} [\mathcal{P}(z_{13}) - \omega(z_1, z_2, z_3) \omega(z_1, z_4, z_3)] + \mathcal{F}_{\mathcal{N}} + (1, 3 \leftrightarrow 2, 4)] \end{aligned} \quad (4.7)$$

plus permutations (3 in all) where

$$\mathcal{F}_{\mathcal{N}=4} = (2\pi)^4 \Lambda_{ab}^\parallel \quad , \quad \mathcal{F}_{\mathcal{N}=2} = -\mathcal{E}_{\mathcal{N}=2} \mathcal{P}(u_{ab}) \quad , \quad \mathcal{F}_{\mathcal{N}=1} = \mathcal{E}_{\mathcal{N}=1} \left[3\eta_1 + \frac{1}{6} \frac{\mathcal{H}'''(0)}{\mathcal{H}'(0)} \right] \quad , \quad (4.8)$$

with $\eta_1 = -2\pi i \partial_\tau \log \eta$.

4.6 Four fermion bilinears, disconnected

$$\begin{aligned} & \left\langle \frac{i}{2} f_{\mu_1 \nu_1}^1 \psi^{\mu_1} \psi^{\nu_1} e^{ip_1 \cdot X}(z_1) \dots \frac{i}{2} f_{\mu_4 \nu_4}^4 \psi^{\mu_4} \psi^{\nu_4} e^{ip_4 \cdot X}(z_4) \right\rangle_{\text{even}}^{\text{disc}} = \\ & \frac{1}{4} (f_1 f_2) (f_3 f_4) \Pi(z_i; p_i) \{ \mathcal{E}_{\mathcal{N}}[\mathcal{P}(z_{12}) + \mathcal{P}(z_{34})] - \mathcal{F}_{\mathcal{N}} \} \end{aligned} \quad (4.9)$$

plus permutations (3 in all).

5. CP odd amplitudes in $\mathcal{N} = 1$ sectors

In the odd spin structure, the presence of a supermodulus requires the insertion of $\delta(\beta) = e^{+\varphi}$ in order to absorb the zero mode of the anti-superghost $\beta = e^{-\varphi} \partial \xi$. The presence of a conformal Killing spinor requires the insertion of $\delta(\gamma) = e^{-\varphi}$ in order to absorb the zero mode of the superghost $\gamma = \eta e^{+\varphi}$. This allows one to fix the position in superspace of one of the vertices that would than be of the form $V = a \cdot \psi \exp(ipX)$. The two combined operations are equivalent to inserting a picture changing operator $\Gamma = e^{\varphi} G + \dots$, where G is the worldsheet supercurrent, at an arbitrary point z_0 and using the (-1) picture for one of the vertices. Independence from z_0 allows one to let z_0 coincide with the position of the vertex in the (-1) picture and replace it with the expression in the (0) picture after using $\langle e^{\varphi(z)} e^{-\varphi(w)} \rangle_{\text{odd}} = 1$. Moreover one has to absorb the four zero-modes of the spacetime fermions present in $\mathcal{N} = 1$ sectors. In $\mathcal{N} = 2$ and $\mathcal{N} = 4$ sectors, CP odd amplitudes with only vector bosons vanish. There is no way to absorb the two (for $\mathcal{N} = 2$) or six (for $\mathcal{N} = 4$) additional zero-modes of the internal fermions present in these sectors. More complicated amplitudes with matter scalars and fermions can accomplish the task.

Let us thus concentrate on $\mathcal{N} = 1$ sectors and start from the simplest non vanishing contribution.

5.1 Two fermion bilinears (6 terms)

Thanks to the exact cancellation between bosonic and fermionic non-zero modes on the worldsheet the final result for terms of the form $\langle \partial X \partial X : \psi \psi : \psi \psi : \rangle_{\text{odd}}$ is very simple and compact

$$\begin{aligned} \langle \partial X(z_1) \partial X(z_2) : \psi \psi : (z_3) : \psi \psi : (z_4) \rangle_{\text{odd}} &= \frac{2}{2^2} \left(\sqrt{\frac{2}{\tau_2^A}} \right)^4 (f_3 \cdot \tilde{f}_4) \mathcal{X}_{ab}^{\mathcal{N}=1} \times \\ & [a_1 \cdot a_2 \partial_1 \partial_2 \mathcal{G}(z_{12}) - \sum_{i \neq 1} a_1 \cdot p_i \partial_1 \mathcal{G}(z_{1i}) \sum_{j \neq 2} a_2 \cdot p_j \partial_2 \mathcal{G}(z_{2j})] \Pi(z_i; p_i) \end{aligned} \quad (5.1)$$

where $\Pi(z_i; p_i)$ denotes the momentum factor (3.25) and the overall coefficient takes into account symmetry factors and the correct normalization of the fermionic zero-modes. In addition there are five more permutations.

5.2 Three fermion bilinears (12 terms)

The next simplest term is

$$\langle : \psi \psi : (z_1) : \psi \psi : (z_2) : \psi \psi : (z_3) \partial X(z_4) \rangle_{\text{odd}} . \quad (5.2)$$

The four zero-modes can be absorbed in three distinct ways. For instance, absorbing two of them at z_3 , one at z_1 and one at z_2 and contracting the remaining two fermions at z_1 and z_2 yield

$$\langle \partial X(\psi\psi)^3 \rangle_{odd} = \frac{2^3}{2^3} \left(\sqrt{\frac{2}{\tau_2^A}} \right)^4 (f_1 \cdot f_2 \cdot \tilde{f}_3) \mathcal{X}_{ab}^{\mathcal{N}=1} \mathcal{S}(z_{12}) \Pi(p_i; z_i) \sum_{i \neq 4} i a_4 \cdot p_i \partial_4 \mathcal{G}(z_{i4}) \quad (5.3)$$

plus two more permutations. In the odd spin structure the fermionic propagator can be taken to be [87, 88]

$$\mathcal{S}(z-w) = -\partial_z \mathcal{G}(z-w) = \partial_z \log \theta_1(z-w) + 2\pi i \frac{\text{Im}(z-w)}{\text{Im}\tau} . \quad (5.4)$$

5.3 Four fermion bilinears (21 terms, 3 structures)

Finally the most laborious term is

$$\langle : \psi\psi : (z_1) : \psi\psi : (z_2) : \psi\psi : (z_3) : \psi\psi : (z_4) \rangle_{odd} . \quad (5.5)$$

In this case there are three possible ways of absorbing zero-modes.

Absorbing two zero-modes at one point (say z_1) and two at another point (say z_2), for a total of 6 permutations, contributes expressions of the form

$$\langle (\psi\psi)^4 \rangle_{odd}^{(2_0 2_0 0_0)} = -\frac{2^2}{2^4} \left(\sqrt{\frac{2}{\tau_2^A}} \right)^4 (f_1 \tilde{f}_2)(f_3 \cdot f_4) \mathcal{X}_{ab}^{\mathcal{N}=1} \mathcal{S}^2(z_{34}) \Pi(p_i; z_i) \quad (5.6)$$

plus permutations.

Next, one can absorb two zero-modes at one point (say z_1), one at another point (say z_2), and one at a third point (say z_3) for a total of 12 permutations contributing expressions of the form

$$\langle (\psi\psi)^4 \rangle_{odd}^{(2_0 1_0 1_0 0)} = \frac{2^4}{2^4} \left(\sqrt{\frac{2}{\tau_2^A}} \right)^4 (f_2 \cdot \tilde{f}_1 \cdot f_3 \cdot f_4) \mathcal{X}_{ab}^{\mathcal{N}=1} \mathcal{S}(z_{24}) \mathcal{S}(z_{34}) \Pi(p_i; z_i) \quad (5.7)$$

plus permutations.

Finally one can absorb one zero mode at each point which yields

$$\langle (\psi\psi)^4 \rangle_{odd}^{(1_0 1_0 1_0 1_0)} = \frac{2^4}{2^4} \left(\sqrt{\frac{2}{\tau_2^A}} \right)^4 \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} f_1^{\mu_1 \nu} f_2^{\mu_1 \nu} f_3^{\mu_3 \rho} f_4^{\mu_4 \rho} \mathcal{X}_{ab}^{\mathcal{N}=1} \mathcal{S}(z_{12}) \mathcal{S}(z_{34}) \Pi(p_i; z_i) , \quad (5.8)$$

as well as two more permutations arising from different Wick contractions of the fermionic non-zero modes.

6. Forward scattering

The recipe for computing string amplitudes requires integrating over the insertion points and then over the modular parameter(s) of the relevant Riemann surface. The task is prohibitively complicated, if not impossible, in general. Yet for some very special amplitudes

or kinematic regimes the situation drastically simplifies. This is the case for non-planar forward scattering that, as we will see, allows to extract interesting predictions for near future colliders.

For forward scattering $p_1 = -p_4$, $p_2 = -p_3$. As a result there is only one non-zero kinematical invariant

$$p_1 \cdot p_2 = p_3 \cdot p_4 = -p_1 \cdot p_3 = p_2 \cdot p_4 = -s/2 = u/2 \quad p_1 \cdot p_4 = p_2 \cdot p_3 = 0 = t/2 \quad (6.1)$$

so that

$$\Pi(p_i, z_i) \rightarrow \Pi(s, z_i) = \exp(s/2(\mathcal{G}_{12} - \mathcal{G}_{13} + \mathcal{G}_{34} - \mathcal{G}_{24})) . \quad (6.2)$$

Moreover, since $a_1 = a_4$ and $a_2 = a_3$, one easily finds

$$(f_1 f_1) = (f_4 f_4) = -(f_1 f_4) = 0 \quad , \quad (f_2 f_2) = (f_3 f_3) = -(f_2 f_3) = 0 \quad (6.3)$$

and

$$(f_1 \tilde{f}_1) = (f_4 \tilde{f}_4) = -(f_1 \tilde{f}_4) = (f_4 \tilde{f}_1) = 0 \quad (6.4)$$

$$(f_2 \tilde{f}_2) = (f_3 \tilde{f}_3) = -(f_2 \tilde{f}_3) = (f_3 \tilde{f}_2) = 0 , \quad (6.5)$$

in addition all cubic contractions vanish

$$(f_i f_j f_k) = 0 \quad , \quad (f_i f_j \tilde{f}_k) = 0 \quad (6.6)$$

since at least two of the f 's are equal (opposite). As a result contractions involving three fermion bilinears give vanishing contribution to forward scattering both to CP even and CP odd processes.

Moreover for non-planar amplitudes the two stacks of branes should be of the same kind $a = b$ so that $T_1 = T_4^\dagger$ and $T_2 = T_3^\dagger$ and $tr_a(T_1 T_2) = tr_b(T_3 T_4)$.

Notice that integration over the four points is unrestricted in the non-planar case, since the Chan-Paton factor $\text{tr}(T_1 T_2) \text{tr}(T_3 T_4)$ is invariant under re-ordering of z_1, z_2 and of z_3, z_4 . So even if *a priori* $0 < z_1 < z_2 < 1$ for a given Chan-Paton factor $\text{tr}(T_1 T_2) \text{tr}(T_3 T_4)$, the other ordering $0 < z_2 < z_1 < 1$ has the same Chan-Paton factor since $\text{tr}(T_1 T_2) = \text{tr}(T_2 T_1)$. This extends immediately to twisted sectors whereby $\text{tr}(T_1 T_2 W^k) \text{tr}(T_3 T_4 W^k)$ is also invariant under reordering since $[W, T_i] = 0$. Integrating by parts is thus possible and further simplifies the non-planar forward scattering amplitudes.

6.1 CP even amplitudes

For forward scattering, dropping total derivatives and Chan-Paton factors but including all relevant permutations, CP even amplitudes with two fermion bilinears read

$$\begin{aligned} & \left\langle \frac{i}{2} f_{\mu_1 \nu_1}^1 \psi^{\mu_1} \psi^{\nu_1} e^{ip_1 \cdot X(z_1)} \frac{i}{2} f_{\mu_2 \nu_2}^2 \psi^{\mu_2} \psi^{\nu_2} e^{ip_2 \cdot X(z_2)} a_2 \cdot \partial_3 X e^{-ip_2 \cdot X(z_3)} a_1 \cdot \partial_4 X e^{-ip_1 \cdot X(z_4)} \right\rangle_{even}^{FS} \\ & + \text{perms} = -\frac{(f_1 f_2)^2}{2\alpha' s} \mathcal{E}_{\mathcal{N}} [\partial_1 \partial_2 \mathcal{G}_{12} + \partial_3 \partial_4 \mathcal{G}_{34} + \partial_1 \partial_3 \mathcal{G}_{13} + \partial_2 \partial_4 \mathcal{G}_{24}] \Pi(z_i; s) , \end{aligned}$$

where, for shortness, $\mathcal{G}_{ij} = \mathcal{G}(z_{ij})$ and

$$(f_1 f_2) = 2[(a_1 \cdot p_2)(a_2 \cdot p_1) - (a_1 \cdot a_2)(p_1 \cdot p_2)] . \quad (6.7)$$

CP even amplitudes arising from connected (box-type) contractions of four fermion bilinears can be conveniently simplified using the identity [87, 88]

$$\mathcal{B}_\alpha(z_1, z_2, z_3, z_4) + \mathcal{B}_\alpha(z_1, z_3, z_2, z_4) + \mathcal{B}_\alpha(z_1, z_3, z_4, z_2) = \frac{1}{2} \partial_z^4 \log \theta_\alpha(z)|_{z=0}, \quad (6.8)$$

where

$$\mathcal{B}_\alpha(z_1, z_2, z_3, z_4) = \mathcal{S}_\alpha(z_{12})\mathcal{S}_\alpha(z_{23})\mathcal{S}_\alpha(z_{34})\mathcal{S}_\alpha(z_{14}) \quad (6.9)$$

and observing that for forward scattering

$$t_1 = (f_1 f_2 f_3 f_4) = (f_1 f_2 f_2 f_1) = (f_1 f_3 f_2 f_4) = a_1^2 a_2^2 (p_1 \cdot p_2)^2 \quad (6.10)$$

and

$$t_2 = (f_1 f_3 f_4 f_2) = (f_1 f_2 f_1 f_2) = \frac{1}{2} (f_1 f_2)^2 = 2[(a_1 \cdot p_2)(a_2 \cdot p_1) - (a_1 \cdot a_2)(p_1 \cdot p_2)]^2. \quad (6.11)$$

One then has

$$\begin{aligned} & \{t_1[\mathcal{B}_\alpha(z_1, z_2, z_3, z_4) + \mathcal{B}_\alpha(z_1, z_3, z_2, z_4)] + t_2 \mathcal{B}_\alpha(z_1, z_3, z_4, z_2)\} \mathcal{Z}_\alpha \\ &= \frac{1}{2} t_1 \mathcal{Z}_\alpha \partial_z^4 \log \theta_\alpha(z)|_{z=0} + (t_2 - t_1) \mathcal{B}_\alpha(z_1, z_3, z_4, z_2) \mathcal{Z}_\alpha. \end{aligned} \quad (6.12)$$

Summing over the spin structures eventually yields

$$\begin{aligned} & \left\langle \frac{i}{2} f_{\mu_1 \nu_1}^1 \psi^{\mu_1} \psi^{\nu_1} e^{ip_1 \cdot X(z_1)} \frac{i}{2} f_{\mu_2 \nu_2}^2 \psi^{\mu_2} \psi^{\nu_2} e^{ip_2 \cdot X(z_2)} \right. \\ & \left. \frac{i}{2} f_{\mu_3 \nu_3}^2 \psi^{\mu_3} \psi^{\nu_3} e^{-ip_2 \cdot X(z_3)} \frac{i}{2} f_{\mu_4 \nu_4}^1 \psi^{\mu_4} \psi^{\nu_4} e^{-ip_4 \cdot X(z_4)} \right\rangle_{even}^{FS, conn} = \\ & \Pi(z_i; s) \{ (t_2 - t_1) \mathcal{E}_\mathcal{N}[\mathcal{P}(z_{14}) - \omega(z_1, z_4, z_2)\omega(z_1, z_4, z_3)] + (t_2 + 2t_1) \mathcal{F}_\mathcal{N} \}. \end{aligned} \quad (6.13)$$

Recall that symmetry under (14) \leftrightarrow (23) exchange is expected and would follow if (3.59) hold.

For forward scattering the disconnected CP even contractions of four fermion bilinears yield

$$\begin{aligned} & \left\langle \frac{i}{2} f_{\mu_1 \nu_1}^1 \psi^{\mu_1} \psi^{\nu_1} e^{ip_1 \cdot X(z_1)} \frac{i}{2} f_{\mu_2 \nu_2}^2 \psi^{\mu_2} \psi^{\nu_2} e^{ip_2 \cdot X(z_2)} \right. \\ & \left. \frac{i}{2} f_{\mu_3 \nu_3}^2 \psi^{\mu_3} \psi^{\nu_3} e^{-ip_2 \cdot X(z_3)} \frac{i}{2} f_{\mu_4 \nu_4}^1 \psi^{\mu_4} \psi^{\nu_4} e^{-ip_4 \cdot X(z_4)} \right\rangle_{even}^{FS, disc} = \\ & \frac{1}{2} \Pi(z_i; s) t_2 \{ \mathcal{E}_\mathcal{N}[\mathcal{P}(z_{12}) + \mathcal{P}(z_{34}) + \mathcal{P}(z_{13}) + \mathcal{P}(z_{24})] - 2\mathcal{F}_\mathcal{N} \}. \end{aligned} \quad (6.14)$$

6.2 CP odd amplitudes

For forward scattering CP odd contractions of two fermion bilinears, after integrating by parts and including all the four non-vanishing permutations, one finds (recall $\alpha' = 1/2$)

$$\begin{aligned} & \langle \partial X \partial X : \psi \psi :: \psi \psi : \rangle_{odd}^{FS} = -\frac{(f_1 f_2)(f_1 \tilde{f}_2)}{4\alpha' s} \left(\sqrt{\frac{2}{\tau_2^A}} \right)^4 \mathcal{X}_{ab}^{\mathcal{N}=1} \times \\ & \Pi(z_i; s) [\partial_1 \partial_2 \mathcal{G}(z_{12}) + \partial_3 \partial_4 \mathcal{G}(z_{34}) + \partial_1 \partial_3 \mathcal{G}(z_{13}) + \partial_2 \partial_4 \mathcal{G}(z_{24})]. \end{aligned} \quad (6.15)$$

For forward scattering CP odd contractions of four fermion bilinears can be simplified by means of the identities

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4} f_1^{\mu_1\nu} \eta_{\nu\lambda} f_2^{\mu_1\lambda} f_1^{\mu_3\rho} \eta_{\rho\sigma} f_2^{\mu_4\sigma} = -\frac{1}{2}(f_1\tilde{f}_2)(f_1f_2) \quad (6.16)$$

and

$$(f_1\tilde{f}_2f_1f_2) = \frac{1}{2}(f_1\tilde{f}_2)(f_1f_2) . \quad (6.17)$$

Thus eventually one gets

$$\langle : \psi\psi : : \psi\psi : : \psi\psi : : \psi\psi : \rangle_{odd}^{FS} = \frac{1}{4}(f_1\tilde{f}_2)(f_1f_2)\mathcal{X}_{ab}^{N=1}(\mathcal{S}_{12} - \mathcal{S}_{34} - \mathcal{S}_{13} + \mathcal{S}_{24})^2 . \quad (6.18)$$

7. Imaginary part and total cross section

According to the optical theorem, the total cross section for the production of closed string states in the bulk, obtains from the imaginary part of the non-planar forward scattering amplitude

$$\sigma_{tot}(s) = \frac{1}{s} Im\mathcal{A}_{FS}(s) . \quad (7.1)$$

It turns out to prove convenient to transform the non-planar amplitude to the transverse channel that exposes the ‘tree-level’ unoriented closed string exchange. It is remarkable but not unexpected that our final compact expressions for the amplitudes transform covariantly, thus providing a check of their validity, if needed. Indeed, performing an S-modular transformation from $\tau = it/2$ to $\tilde{\tau} = i\ell$ one finds

$$\mathcal{F}_{\mathcal{N}}(\tau = -1/\tilde{\tau}) = -i\tilde{\tau}^3\mathcal{F}_{\mathcal{N}}(\tilde{\tau}) \quad (7.2)$$

for all the correlators we have computed in any sector of the theory. The overall power of $\tilde{\tau}$ then cancels against the measure of integration $dt/t = d\ell/\ell$. Under the required S-modular transformation, the combinations $u_{ab}^I = kv_{ab}^I + \varepsilon_{ab}^I\tau_{\mathcal{A}}$ transform into $\tilde{u}_{ab}^I = kv_{ab}^I\tilde{\tau}_{\mathcal{A}} - \varepsilon_{ab}^I$. What was a projection in the direct channel becomes a mass-shift in the transverse channel and *vice versa*. The boundary insertion points z_1 and z_2 gets re-located onto a unit segment along the real axis, while z_3 and z_4 gets re-located onto a unit segment parallel to the real axis and displaced from it by an amount $\tilde{\tau}_2/2 = \ell/2$.

Thanks to the symmetry of the Chan-Paton factors, the integration is unrestricted and total derivatives can be dropped since there is no boundary contribution. Indeed, terms of the form

$$\partial_1[\mathcal{G}(z_{13}) - \mathcal{G}(z_{12})]\partial_4[\mathcal{G}(z_{42}) - \mathcal{G}(z_{43})]\Pi(s, z_i) = (2/s)^2\partial_1\partial_4\Pi(s, z_i) \quad (7.3)$$

being total derivatives integrate to zero. The relative sign appear due to $p_3 = -p_2$. Similarly for 2, 3 since $p_4 = -p_1$.

Terms of the form

$$\partial_1[\mathcal{G}(z_{12}) - \mathcal{G}(z_{13})]\partial_2[\mathcal{G}(z_{21}) - \mathcal{G}(z_{24})]\Pi(s, z_i) \quad (7.4)$$

are more involved. One has

$$\begin{aligned} \partial_1[\mathcal{G}(z_{12}) - \mathcal{G}(z_{13})]\partial_2[\mathcal{G}(z_{21}) - \mathcal{G}(z_{24})]\Pi(s, z_i) &= (2/s)\partial_1[\mathcal{G}(z_{12}) - \mathcal{G}(z_{13})]\partial_2\Pi(s, z_i) \\ &= (2/s)\partial_2\{\partial_1[\mathcal{G}(z_{12}) - \mathcal{G}(z_{13})]\Pi(s, z_i)\} - (2/s)\partial_2\partial_1[\mathcal{G}(z_{12}) - \mathcal{G}(z_{13})]\Pi(s, z_i). \end{aligned} \quad (7.5)$$

The first term is a total derivative and integrates to zero. The second term can be conveniently rewritten as

$$\partial_2\partial_1\mathcal{G}(z_{12})\Pi(s, z_i) = -\frac{\alpha'}{2}[\mathcal{P}(z_{12}) + 2\eta_1 + \frac{\pi}{2\tau_2^A}]\Pi(s, z_i). \quad (7.6)$$

The same applies to the pairs of points (1, 3), (4, 3) and (2, 4).

Dropping all the tildes for simplicity, the final form of the worldsheet integrals one needs to compute for the CP even case is

$$\begin{aligned} \mathcal{A}_{\mathcal{N}}^{FS}(s) &= \int d\ell \mathcal{X}_{\mathcal{N}} \int \prod_i dz_i e^{\frac{s}{2}(\mathcal{G}_{12} - \mathcal{G}_{13} + \mathcal{G}_{34} - \mathcal{G}_{24})} \{2t_1 \mathcal{F}_{\mathcal{N}} + \frac{t_2}{\alpha' s} \mathcal{E}_{\mathcal{N}} \left(4\eta_1 + \frac{\pi}{\ell}\right) + \\ &+ (t_2 - t_1) \mathcal{E}_{\mathcal{N}}[\mathcal{P}_{14} - (\partial_1 \mathcal{G}_{14})^2] + \frac{t_2}{2} \frac{\alpha' s + 1}{\alpha' s} \mathcal{E}_{\mathcal{N}}[\mathcal{P}_{12} + \mathcal{P}_{34} + \mathcal{P}_{13} + \mathcal{P}_{24}]\}. \end{aligned} \quad (7.7)$$

For $\mathcal{N} = 4$ sectors, $\mathcal{E}_{\mathcal{N}=4} = 0$ and only the first term contributes and yields

$$\mathcal{A}_{\mathcal{N}=4}^{FS} = K_{\mathcal{N}=4}(s; a_1, a_2) \int_0^\infty d\ell \frac{\tilde{\Lambda}_{6-\hat{d}}(\ell)}{\ell^{\hat{d}/2}} \prod_i \int_0^1 dx_i \Pi_X(\ell; z_i = x_i + i\delta_i), \quad (7.8)$$

where

$$K_{\mathcal{N}=4}(s; a_1, a_2) = \frac{2a_1^2 a_2^2 (p_1 \cdot p_2)^2 (2\pi)^4 \hat{V}_{6-\hat{d}}}{2_{GSO} 2_\Omega 2_{P_4}^2 2_{\Lambda_{6-\hat{d}}}^3 N_{orb}} \text{tr}(T_1 T_2)^2, \quad (7.9)$$

$0 \leq \hat{d} \leq 6$ is the number of ‘large’ internal dimensions in the $D3$ -brane description, i.e. ‘small’ in the T-dual $D9$ -brane description, and $\delta_1 = \delta_2 = 0$ while $\delta_3 = \delta_4 = \ell/2$. Dependence on the insertion points is only through $\Pi_X(\ell; z_i)$.

Exploiting the series expansions in $q = e^{-2\pi\ell}$ collected in an appendix, one finds

$$\begin{aligned} [4q^{1/4} \sin(\pi x_{12}) \sin(\pi x_{34})]^{\alpha' s} \Pi_X(q) &= 1 - 2\alpha' s q^{1/2} [\cos(2\pi x_{13}) + \cos(2\pi x_{24})] \\ &+ 2\alpha' s q \{1 + \cos(2\pi x_{12}) + \cos(2\pi x_{34}) + \\ &(\alpha' s - 1) [\cos^2(2\pi x_{13}) + \cos^2(2\pi x_{24})] + 2\alpha' s \cos(2\pi x_{13}) \cos(2\pi x_{24})\} \\ &+ \frac{4}{3} \alpha' s q^{3/2} \{ [1 + \cos(2\pi x_{12}) + \cos(2\pi x_{34}) + (\alpha' s - 1) \cos(2\pi x_{13}) \cos(2\pi x_{24})] \times \\ &\times 3\alpha' s [\cos(2\pi x_{13}) + \cos(2\pi x_{24})] + (\alpha' s - 1)(\alpha' s - 2) [\cos^3(2\pi x_{13}) + \cos^3(2\pi x_{24})] \} \\ &+ \frac{1}{3} \alpha' s q^2 \{ 2(\alpha' s - 1)(\alpha' s - 2)(\alpha' s - 3) [\cos^4(2\pi x_{13}) + \cos^4(2\pi x_{24})] + \\ &+ 8\alpha' s \cos(2\pi x_{13}) \cos(2\pi x_{24}) [(\alpha' s - 1)(\alpha' s - 2)(\cos^2(2\pi x_{13}) + \cos^2(2\pi x_{24})) + \\ &3\alpha' s (1 + \cos(2\pi x_{12}) + \cos(2\pi x_{34}))] + \\ &+ 12\alpha' s (\alpha' s - 1)^2 \cos^2(2\pi x_{13}) \cos^2(2\pi x_{24}) + \\ &+ 12[\cos^2(2\pi x_{13}) + \cos^2(2\pi x_{24})] (1 + \alpha' s (\alpha' s - 1) [1 + \cos(2\pi x_{12}) + \cos(2\pi x_{34})]) \} \end{aligned}$$

$$\begin{aligned}
 &+3\{2(\alpha's + 1)[\cos(2\pi x_{12}) + \cos(2\pi x_{34})]^2 - 4\cos(2\pi x_{12})\cos(2\pi x_{34}) + \\
 &+2(2\alpha's + 1)[\cos(2\pi x_{12}) + \cos(2\pi x_{34})] + 2\alpha's - 3\} + \dots .
 \end{aligned}
 \tag{7.10}$$

Truncating to lowest order (i.e. q^0), performing the trigonometric integrals over the insertion points by means of⁵

$$\int_0^1 dx (\sin \pi x)^a (\cos \pi x)^{2n} = \frac{\Gamma(\frac{1+a}{2}) \Gamma(\frac{1}{2} + n)}{\pi \Gamma(1 + n + \frac{a}{2})},
 \tag{7.11}$$

and extracting the imaginary part by means of

$$\text{Im} \left(\int_0^\infty d\ell \ell^{-\alpha} e^{-\beta\ell} \right) = \frac{\pi \beta^{\alpha-1}}{\Gamma(\alpha)},
 \tag{7.12}$$

one gets

$$\sigma_0(s) = K_{\mathcal{N}=4}(s; a_1, a_2) \frac{\pi}{s \Gamma(\frac{d}{2})} \left(-\frac{\pi \alpha's}{2} \right)^{\frac{d}{2}-1} \left[\frac{\Gamma(\frac{1-\alpha's}{2})}{2^{\alpha's} \sqrt{\pi} \Gamma(1 - \frac{\alpha's}{2})} \right]^2
 \tag{7.13}$$

for $0 \leq \alpha's < 4$, in perfect agreement with the results of [71], *mutatis mutandis*.

Integration over x_{13} (equivalently x_{24}) effectively kills all half odd integer powers of q in the expansion of Π_X , given in an appendix. The next contribution in $\mathcal{N} = 4$ sectors thus comes from terms of order q^1 . Performing the trigonometric integrals over the insertion points by means of (7.11) and extracting the imaginary part yields

$$\sigma_1(s) = K_{\mathcal{N}=4}(s; a_1, a_2) \frac{\pi}{s \Gamma(\frac{d}{2})} \left(-\frac{\pi(\alpha's - 4)}{2} \right)^{\frac{d}{2}-1} \left[\frac{\Gamma(\frac{1-\alpha's}{2})}{2^{\alpha's} \sqrt{\pi} \Gamma(1 - \frac{\alpha's}{2})} \right]^2 2B_1(\alpha's)
 \tag{7.14}$$

for $4 \leq \alpha's < 8$, where the 'form factor' is given by

$$B_1(\alpha's) = 2(\alpha's)^2 \frac{(\alpha's)^2 - 3\alpha's + 4}{(\alpha's - 2)^2} = 2(\alpha's)^2 \frac{(\alpha's - \frac{3}{2})^2 + \frac{7}{4}}{(\alpha's - 2)^2},
 \tag{7.15}$$

in perfect agreement with the results of [71], *mutatis mutandis*.

For $\mathcal{N} = 2$ sectors the last term $\mathcal{F}_{\mathcal{N}=2}$ gives similar results after replacing the overall kinematical factor with

$$K_{\mathcal{N}=2}(s; a_1, a_2) = \frac{\pi^2 2a_1^2 a_2^2 (p_1 \cdot p_2)^2 (2\pi)^2 \mathcal{I}_{ab}^\perp \hat{V}_{2-\hat{d}}}{3 \cdot 2_{GSO} 2_\Omega 2_{P_4}^2 2_{\Lambda_{2-\hat{d}}}^\parallel N_{orb}} \text{tr}(W_k T_1 T_2)^2,
 \tag{7.16}$$

⁵It is easy too see that $\int_0^1 dx (\sin \pi x)^a (\cos \pi x)^{2n+1} = 0$ or more generally

$$\int_0^1 dx (\sin \pi x)^a (\cos \pi x)^b = \frac{1 + e^{i\pi b}}{2\pi} \frac{\Gamma(\frac{1+a}{2}) \Gamma(\frac{1+b}{2})}{\Gamma(1 + \frac{a+b}{2})}$$

where the first factor comes from the constant term (q^0) in the expansion of $-\mathcal{P}(u)$, and taking into account that the number of ‘large’ internal dimensions satisfies $0 \leq \hat{d} \leq 2$ in these sectors.

For $\mathcal{N} = 1$ sectors the last term $\mathcal{F}_{\mathcal{N}=1}$ gives poles rather than a cut since $\hat{d} = 0$ in these sectors. The massless poles corresponds to the dependence of the gauge couplings on the VEV’s of the massless closed string scalars in the twisted sectors. Massive poles signal the possibility of producing (unstable) closed string i.e. one should read ‘resonances’ with not necessarily integer masses (in $1/\alpha'$ units).

A potential negative mass pole might arise from $\mathcal{P}(z_{12})$ and similar terms. By OPE considerations however it should be absent. Reassuringly one finds this kind of terms with coefficient proportional to $1 + (1/\alpha' s)$ so that

$$\left(1 + \frac{1}{\alpha' s}\right) \int_0^1 (\sin \pi x)^{-\alpha' s - 2} dx = \frac{\Gamma(\frac{1-\alpha' s}{2})}{2\alpha' s \sqrt{\pi} \Gamma(1 - \frac{\alpha' s}{2})} \quad (7.17)$$

has no ‘tachyonic’ pole. The next term in the q expansion of $P(z_{12})$ is a constant ($-\pi^2/3$) that yields integrals of the form (7.13). i.e. the form factor of the ‘zero’ mass states. The term of order q is proportional to $\sin^2(2\pi x)$. Integration then yields

$$\int_0^1 (\sin \pi x)^{-\alpha' s + 2} dx = \frac{\alpha' s - 1}{\alpha' s - 2} \frac{\Gamma(\frac{1-\alpha' s}{2})}{2\alpha' s \sqrt{\pi} \Gamma(1 - \frac{\alpha' s}{2})}. \quad (7.18)$$

The same situation prevails for the terms in $P(z_{34})$.

Finally one should consider the combinations $\mathcal{P}(z_{14}) - (\partial_1 \mathcal{G}_{14})^2$, plus the corresponding ones with (1, 4) replaced by (2, 3). Quite remarkably

$$\mathcal{P}(z_{14}) - (\partial_1 \mathcal{G}_{14})^2 = -4\pi i \frac{\partial}{\partial \tau} \log \left(\frac{\theta_4(x_{14}|\tau)}{\eta(\tau)} \right) \quad (7.19)$$

that admits the following expansion

$$\begin{aligned} \mathcal{P}(z_{14}) - (\partial_1 \mathcal{G}_{14})^2 &= -\frac{\pi^2}{3} - 8\pi^2 \sum_{n, d_n|n} \frac{1 - (-)^{n/d_n}}{2} q^{n/2} \frac{n}{d_n} \cos(2\pi d_n x_{14}) \\ &= -\frac{\pi^2}{3} - 8\pi^2 [q^{1/2} \cos(2\pi x_{14}) + q \cos(4\pi x_{14}) + \dots], \end{aligned} \quad (7.20)$$

where $d_n|n$ denotes the divisors of n . Combining with Π_X one finds that half odd integer powers of q vanish after integration over dx_{13} or, equivalently, dx_{24} . However new integer powers are generated by combinations of integer and half odd integer powers of q Π_X and $\mathcal{P}(z_{14}) - (\partial_1 \mathcal{G}_{14})^2$. For instance at order q one finds

$$\begin{aligned} &-\frac{2\pi^2 \alpha' s}{3} \{1 + \cos(2\pi x_{12}) + \cos(2\pi x_{34}) + \\ &(\alpha' s - 1)[\cos^2(2\pi x_{13}) + \cos^2(2\pi x_{24})] + 2\alpha' s \cos(2\pi x_{13}) \cos(2\pi x_{24})\} \\ &-8\pi^2 \cos(4\pi x_{14}) + 16\pi^2 \alpha' s \cos(2\pi x_{14}) [\cos(2\pi x_{13}) + \cos(2\pi x_{24})]. \end{aligned} \quad (7.21)$$

After integration over dx_{13} or, equivalently, dx_{24} , the last term gives $8\pi^2\alpha's[\cos(2\pi x_{34}) + \cos(2\pi x_{12})]$, that modifies $\sigma_1(s)$ when combined with the lowest order term in⁶

$$\mathcal{E}_{\mathcal{N}=1} \approx 3\pi + 2\pi \sum_I q^{kv_{ab}^I} + 4\pi \sum_I q^{1-kv_{ab}^I} + \dots \quad (7.22)$$

New thresholds with fractional mass appear due to the fractional powers in the expansion of $\mathcal{E}_{\mathcal{N}}$.

Terms in $\mathcal{P}(z_{13})$ and $\mathcal{P}(z_{24})$ can be discussed similarly.

8. Comments

In their present form, our results are not directly related to processes observable at LHC. Without some recoiling observable (open string) states it is impossible to detect the decay into closed strings in the bulk. Yet it should not be difficult to include some soft observable particle along the lines of [71]. For hadronic colliders, such as LHC, a much subtler issue is how to extract hadronic cross sections from the ‘partonic’ cross sections we have computed. One has to convolute our or similar results with the partonic distributions of the relevant hadrons, i.e. the proton. To the best of our knowledge these are not known in analytic form but significant effort [89] is presently devoted into this important step.

At a more formal level, our results, obtained for a specific yet interesting class of supersymmetric models with open and unoriented strings, display a remarkably simple structure. This is largely due to the already observed fact that open string gauge bosons belong to the ‘identity’ sector of the internal conformal field theory, describing the compactification from $D = 10$. We thus see no major obstacle in extending them to the case of genuinely interacting internal $\mathcal{N} = 2$ SCFT’s, such as Gepner models [90]. It is tantalizing to speculate that

$$\mathcal{E}^{(s)} = - \sum_{\alpha} c_{\alpha} e_{\alpha-1} \mathcal{Z}_{\alpha}^{(s)} \quad (8.1)$$

and

$$\mathcal{F}^{(s)} = \sum_{\alpha} c_{\alpha} e_{\alpha-1}^2 \mathcal{Z}_{\alpha}^{(s)} \quad (8.2)$$

should remain valid, once the relevant partition functions $\mathcal{Z}_{\alpha}^{(s)}$, with s ranging over all the sectors of the open string spectrum, are extracted from the supersymmetric characters

$$\chi_{SUSY}^{(s)} = \sum_{\alpha} c_{\alpha} \mathcal{Z}_{\alpha}^{(s)} = \sum_{\alpha} c_{\alpha} \frac{\theta_{\alpha}(0)}{\eta^3} \mathcal{W}_{\alpha}^{(s)}, \quad (8.3)$$

where $\mathcal{W}_{\alpha}^{(s)}$ denotes the contribution of the sector s of the internal $\mathcal{N} = 2$ SCFT in a given spin structure α [91, 92].

We would like to conclude with a comment on the supersymmetry properties of our amplitudes. Some time ago [93], Berkovits and Vallilo have proposed manifestly supersymmetric one-loop amplitudes for massless closed string states based on the hybrid formalism [94]. Deducing similar amplitudes for massless open string states should be straightforward in the hybrid formalism. However due to the factorization of the spacetime and

⁶For simplicity we assume $kv_{ab}^I < 1$.

internal SCFT's in the hybrid formalism, in the absence of RR fluxes, it is not clear to us how to reproduce the simple yet non trivial internal structures, such as the functions $\mathcal{E}_{\mathcal{N}=1}$ and $\mathcal{F}_{\mathcal{N}=1}$ that we have found in the NSR formalism. Other manifestly supersymmetric formalisms [67, 95] may help clarifying this issue. We leave it to future work with eyes wide open to the possibilities of dealing with RR fluxes [96] and the associated non-trivial warping arising in flux compactifications [97].

Acknowledgments

We would like to thank Ya. Stanev, E. Trevigne, A. Kumar, M. Serone, E. Kiritsis, I. Klebanov for useful discussions. During completion of this work M.B. was visiting the Galileo Galilei Institute of Arcetri (FI), INFN is acknowledged for hospitality and hopefully for support. M.B. would also like to thank the organizers (C. Angelantonj, E. Dudas, T. Gherghetta and A. Pomarol) and the participants to the workshop “Beyond the Standard Model”, especially R. Blumenhagen, for creating a stimulating environment on the expectations for LHC.⁷ This work was supported in part by INFN, by the MIUR-COFIN contract 2003-023852, by the EU contracts MRTN-CT-2004-503369 and MRTN-CT-2004-512194, by the INTAS contract 03-516346 and by the NATO grant PST.CLG.978785.

A. Elliptic functions

A.1 Definitions

Let $q = e^{2\pi i\tau}$ the Jacobi θ -functions are defined as gaussian sums

$$\theta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} (z|\tau) = \sum_n q^{\frac{1}{2}(n+\alpha)^2} e^{2\pi i(z-\beta)(n-\alpha)}, \quad (\text{A.1})$$

where $\alpha, \beta \in \mathbb{R}$.

Equivalently, for particular values of characteristics, such as $\alpha, \beta = 0, 1/2$ they are given also in terms of infinite product as follows

$$\begin{aligned} \theta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} (z|\tau) &= \theta_1(z|\tau) = 2q^{\frac{1}{8}} \sin(\pi z) \prod_{m=1}^{\infty} (1 - q^m)(1 - e^{2\pi iz} q^m)(1 - e^{-2\pi iz} q^m) \\ \theta \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} (z|\tau) &= \theta_2(z|\tau) = 2q^{\frac{1}{8}} \cos(\pi z) \prod_{m=1}^{\infty} (1 - q^m)(1 + e^{2\pi iz} q^m)(1 + e^{-2\pi iz} q^m) \\ \theta \begin{bmatrix} 0 \\ 0 \end{bmatrix} (z|\tau) &= \theta_3(z|\tau) = \prod_{m=1}^{\infty} (1 - q^m)(1 + e^{2\pi iz} q^{m-\frac{1}{2}})(1 + e^{-2\pi iz} q^{m-\frac{1}{2}}) \\ \theta \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} (z|\tau) &= \theta_4(z|\tau) = \prod_{m=1}^{\infty} (1 - q^m)(1 - e^{2\pi iz} q^{m-\frac{1}{2}})(1 - e^{-2\pi iz} q^{m-\frac{1}{2}}). \end{aligned} \quad (\text{A.2})$$

Dedekind η function is defined as

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n) \quad (\text{A.3})$$

⁷The most credited version being now: **L'italia Ha vinto il Campionato del mondo.**

and satisfies $\theta_1'(0) = 2\pi\eta^3$.

Weierstrass \mathcal{P} function

$$\mathcal{P}(z) = -\partial_z^2 \log \theta_1(z) - 2\eta_1, \quad (\text{A.4})$$

where

$$\eta_1 = -2\pi i \frac{\partial}{\partial \tau} \log \eta = -\frac{1}{6} \frac{\theta_1'''(0)}{\theta_1'(0)}, \quad (\text{A.5})$$

has a double pole at $z = 0$ and is bi-periodic in z .

The free fermionic propagator in the even spin structures (Szego kernel)

$$\mathcal{S}_\alpha(z) = \frac{\theta_\alpha(z) \theta_1'(0)}{\theta_1(z) \theta_\alpha'(0)} \quad (\text{A.6})$$

has a simple pole at $z = 0$ and satisfies

$$\mathcal{S}_\alpha(z)^2 = \mathcal{P}(z) - e_{\alpha-1}, \quad (\text{A.7})$$

where

$$e_{\alpha-1} = -4\pi i \frac{\partial}{\partial \tau} \log \frac{\theta_\alpha(0)}{\eta} \quad (\text{A.8})$$

are also related to $\mathcal{P}(z)$ evaluated at the semiperiods

$$e_1 = \mathcal{P}\left(\frac{1}{2}\right), \quad e_2 = \mathcal{P}\left(\frac{1+\tau}{2}\right), \quad e_3 = \mathcal{P}\left(\frac{\tau}{2}\right). \quad (\text{A.9})$$

In the odd spin structure

$$\mathcal{S}_1(z) = -\partial_z \mathcal{G}(z) \quad (\text{A.10})$$

is biperiodic with a simple pole but not analytic.

The free bosonic propagator (biperiodic with logarithmic behaviour at $z = 0$) on the (covering) torus is given by

$$\mathcal{G}(z) = -\frac{\alpha'}{2} \left[\log \left| \frac{\theta_1(z)}{\theta_1'(0)} \right|^2 - \frac{2\pi}{Im\tau} Im z^2 \right]. \quad (\text{A.11})$$

A.2 Pseudo-periodicity and zeroes

Under lattice shifts of their first argument z , theta functions transform according to

$$\theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (z + 1|\tau) = e^{2\pi i \alpha} \theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (z|\tau) \quad (\text{A.12})$$

$$\theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (z + \tau|\tau) = e^{-2\pi i(z+\beta) - i\pi\tau} \theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (z|\tau). \quad (\text{A.13})$$

The location of their zeroes is given by

$$\theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (z|\tau) = 0 \quad \leftrightarrow \quad z_{n,m} = \left(\alpha - \frac{1}{2} + n\right)\tau + \left(\beta - \frac{1}{2} + m\right). \quad (\text{A.14})$$

A.3 Modular Transformations

Under T and S modular transformations of their arguments theta functions transform according to

$$\begin{aligned}
 \theta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} (z|\tau + 1) &= e^{-i\pi\alpha(\alpha-1)} \theta \begin{bmatrix} \alpha \\ \beta+\alpha-\frac{1}{2} \end{bmatrix} (z|\tau) \\
 \eta(\tau + 1) &= e^{\frac{i\pi}{12}} \eta(\tau) \\
 \theta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(\frac{z}{\tau} \middle| -\frac{1}{\tau}\right) &= (-i\tau)^{\frac{1}{2}} e^{2i\pi\alpha\beta+i\pi z^2/\tau} \theta \begin{bmatrix} \beta \\ -\alpha \end{bmatrix} (z|\tau) \\
 \eta\left(-\frac{1}{\tau}\right) &= (-i\tau)^{\frac{1}{2}} \eta(\tau) .
 \end{aligned} \tag{A.15}$$

The modular transformation P, that connects the direct and transverse channel of Möbius strip amplitudes, is more involved. It consists in a sequence of T and S transformations ($P = TST^2S$) on the modular parameter $\tau_M = \frac{1}{2} + \frac{it}{2}$

$$\begin{aligned}
 \theta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(\frac{z}{it} \middle| \frac{1}{2} + \frac{i}{2t}\right) &= e^{-i\pi\alpha(\alpha-1)-2\pi i(\alpha+\beta-1/2)^2+2\pi z^2/t} \sqrt{-it} \theta \begin{bmatrix} \alpha+2\beta-2 \\ 1/2-\alpha-\beta \end{bmatrix} \left(z \middle| \frac{1}{2} + \frac{it}{2}\right) \\
 \eta\left(\frac{1}{2} + \frac{i}{2t}\right) &= e^{i\pi/4} \sqrt{-it} \eta\left(\frac{1}{2} + \frac{it}{2}\right) .
 \end{aligned} \tag{A.16}$$

A.4 Useful Identities

Riemann identity for even spin structures reads

$$\begin{aligned}
 \sum_{\alpha} c_{\alpha} \theta_{\alpha}(z_1) \theta_{\alpha}(z_2) \theta_{\alpha}(z_3) \theta_{\alpha}(z_4) &= \\
 \theta_1(z'_1) \theta_1(z'_2) \theta_1(z'_3) \theta_1(z'_4) &- \theta_1(z''_1) \theta_1(z''_2) \theta_1(z''_3) \theta_1(z''_4) ,
 \end{aligned} \tag{A.17}$$

where z'_i and z''_i are related to z_i through

$$\begin{aligned}
 z'_1 &= \frac{1}{2}(z_1 + z_2 + z_3 + z_4) & z'_2 &= \frac{1}{2}(z_1 + z_2 - z_3 - z_4) \\
 z'_3 &= \frac{1}{2}(z_1 - z_2 + z_3 - z_4) & z'_4 &= \frac{1}{2}(z_1 - z_2 - z_3 + z_4)
 \end{aligned} \tag{A.18}$$

and

$$\begin{aligned}
 z''_1 &= \frac{1}{2}(-z_1 + z_2 + z_3 + z_4) & z''_2 &= \frac{1}{2}(z_1 - z_2 + z_3 + z_4) \\
 z''_3 &= \frac{1}{2}(z_1 + z_2 - z_3 + z_4) & z''_4 &= \frac{1}{2}(z_1 + z_2 + z_3 - z_4) .
 \end{aligned} \tag{A.19}$$

A.5 Series Expansions

Series expansion in powers of q yield

$$\begin{aligned}
 \partial_z \log \theta_1(z, q) &= \pi \cot(\pi z) + 4\pi \sum_n \frac{q^n}{1-q^n} \sin(2\pi n z) \\
 &= \pi \coth(\pi z) + 4\pi \sum_{n, d_n|n} q^n \sin(2\pi d_n z)
 \end{aligned} \tag{A.20}$$

and

$$\begin{aligned} \partial_z^2 \log \theta_1(z, q) &= -\frac{\pi^2}{\sin(\pi z)^2} + 8\pi^2 \sum_n \frac{nq^n}{1-q^n} \cos(2\pi n z) \\ &= -\frac{\pi^2}{\sin(\pi z)^2} + 8\pi^2 \sum_{n, d_n|n} q^n d_n \cos(2\pi d_n z), \end{aligned} \quad (\text{A.21})$$

where, using $\partial_\tau q = 2\pi i q$,

$$\eta_1 \equiv -2\pi i \partial_\tau \log \eta = \frac{\pi^2}{6} - 4\pi^2 \sum_n \frac{nq^n}{1-q^n} = \frac{\pi^2}{6} - 4\pi^2 \sum_{n, d_n|n} q^n d_n \quad (\text{A.22})$$

so that

$$\mathcal{P}(z) \equiv -\partial_z^2 \log \theta_1(z, q) - 2\eta_1 = \frac{\pi^2}{\sin(\pi z)^2} - \frac{\pi^2}{3} + 8\pi^2 \sum_{n, d_n|n} q^n d_n [1 - \cos(2\pi d_n z)]. \quad (\text{A.23})$$

Moreover

$$4\pi i \partial_\tau \log \theta_1(z, q) = -\pi^2 + 8\pi^2 \sum_{n, d_n|n} q^n \frac{n}{d_n} [1 + 2 \cos(2\pi d_n z)] \quad (\text{A.24})$$

and

$$\begin{aligned} \left(\frac{\theta_1'(z, q)}{\theta_1(z, q)} \right)^2 &= \partial_z^2 \log \theta_1(z, q) - \frac{\theta_1''(z, q)}{\theta_1(z, q)} = \partial_z^2 \log \theta_1(z, q) - 4\pi i \partial_\tau \log \theta_1(z, q) \\ &= -\pi^2 \cot(\pi z)^2 - 8\pi^2 \sum_{n, d_n|n} q^n \left[\left(\frac{2n}{d_n} - d_n \right) \cos(2\pi d_n z) + \frac{n}{d_n} \right]. \end{aligned} \quad (\text{A.25})$$

For points on different boundaries in the transverse channel θ_1 gets effectively replaced by θ_4 for which

$$\begin{aligned} \partial_z \log \theta_4(z, q) &= 4\pi \sum_n \frac{q^{n/2}}{1-q^n} \sin(2\pi n z) \\ &= 4\pi \sum_{n, d_n|n} q^{n/2} \frac{1 - (-)^{n/d_n}}{2} \sin(2\pi d_n z) \end{aligned} \quad (\text{A.26})$$

and

$$\begin{aligned} \partial_z^2 \log \theta_4(z, q) &= 8\pi^2 \sum_n \frac{nq^{n/2}}{1-q^n} \cos(2\pi n z) \\ &= 8\pi^2 \sum_{n, d_n|n} q^{n/2} \frac{1 - (-)^{n/d_n}}{2} d_n \cos(2\pi d_n z). \end{aligned} \quad (\text{A.27})$$

Moreover

$$\begin{aligned}
 4\pi i \partial_\tau \log \theta_4(z, q) &= 8\pi^2 \sum_n \left[\frac{nq}{1-q^n} + \frac{2q^n \cos(2\pi n z)}{(1-q^{2n})^2} \right] \\
 &= 8\pi^2 \sum_{n, d_n|n} \left[d_n q^n + \frac{1 - (-)^{n/d_n}}{2} \frac{n}{d_n} q^{n/2} \cos(2\pi d_n z) \right]
 \end{aligned}
 \tag{A.28}$$

so that

$$\begin{aligned}
 \left(\frac{\theta'_4(z, q)}{\theta_4(z, q)} \right)^2 &= \partial_z^2 \log \theta_4(z, q) - \frac{\theta''_4(z, q)}{\theta_4(z, q)} = \partial_z^2 \log \theta_4(z, q) - 4\pi i \frac{\partial_\tau \theta_4(z, q)}{\theta_4(z, q)} \\
 &= -8\pi^2 \sum_{n, d_n|n} \left[d_n q^n + \frac{1 - (-)^{n/d_n}}{2} \left(\frac{n}{d_n} - d_n \right) q^{n/2} \cos(2\pi d_n z) \right].
 \end{aligned}
 \tag{A.29}$$

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